The kinetic energy before gas expulsion can be expressed as:

$$T_1 = \frac{\kappa}{2} M_* \sigma^2 \tag{1}$$

where the index 1 refers to before gas expulsion, M_* is the mass in stars and σ is the velocity dispersion of the stars. To account for the velocity distribution κ is a correction factor which amounts to $\kappa = 1.17$ for a Maxwellian distribution.

A crude approximation for the potential energy of the stars before gas expulsion can be written as:

$$W_1 \approx \frac{G M_{\rm tot}}{R} \cdot M_* \tag{2}$$

where $M_{\rm tot}$ is the total mass (stars and gas), i.e. $M_* = {\rm LSF} \cdot M_{\rm tot}$, LSF being the local stellar fraction, and R is an arbitrary radius, e.g. the half-light radius. Therefore we can express W_1 in terms of M_* as

$$W_1 \approx \frac{G M_*^2}{R {\rm LSF}}. \tag{3}$$

Directly after gas expulsion we have the situation that the velocities of the stars have not changed yet and we get:

$$T_2 = T_1. \tag{4}$$

But with the gas gone the potential energy of the stars is due to the stars alone and we can use the approximation:

$$W_2 \approx \frac{G M_*^2}{R} = {\rm LSF} \cdot W_1. \tag{5}$$

Now we see that our ansatz does not depend on the choice of R.

The virial ratio at (before) gas expulsion can be expressed as:

$$\begin{aligned} Q_{\rm f} &= \frac{T_1}{W_1} = \frac{T_1 {\rm LSF}}{W_2} \\ &= \frac{\kappa M_* \sigma^2 {\rm LSF}}{2 W_2}. \end{aligned} \tag{6}$$

We can express the velocity dispersion of the stars now as

$$\sigma^2 = \frac{2 Q_{\rm f} W_2}{\kappa M_* {\rm LSF}}. \tag{7}$$

At the same time (just after gas expulsion) the escape velocity of the stars is:

$$v_{\rm esc}^2 = 2\Phi_2 = \frac{2W_2}{M_*}. \tag{8}$$

Now we can express the ratio between these two quantities as:

$$\begin{aligned}\frac{\sigma^2}{v_{\rm esc}^2} &= \frac{2Q_{\rm f}W_2M_*}{\kappa M_*{\rm LSF}2W_2} \\ &= \frac{Q_{\rm f}}{\kappa\ {\rm LSF}}.\end{aligned} \tag{9}$$

If we now assume a Maxwellian velocity dispersion and furthermore assume that all remaining bound stars, i.e. $f_{\rm bound}$, have velocities below the escape velocity we get:

$$f_{\rm bound} = F(< X) = {\rm erf}\left(\frac{1}{\sqrt{2}}X\right) - \sqrt{\frac{2}{\pi}}X\exp\left(-\frac{1}{2}X^2\right) \tag{10}$$

$$X = 1.6\frac{\sigma}{v_{\rm esc}} = 1.6\sqrt{\frac{Q_{\rm f}}{\kappa\ {\rm LSF}}}. \tag{11}$$

Even though this looks like a very crude approximation the resulting curves fit the results of our simulations very well. In case of very low bound fractions one might want to think to include a factor $f_{\rm bound}$ in the calculation of the potential, i.e. stars with velocities higher than the escape velocity are not contributing to the potential and solve the equations iteratively.