Algunos Conceptos para intender Interferometria

1. Transforme de Fourier

2. Young's Double Slit Experiment

3. plano de *uv*

Algunos Concepctos del coordinados

Fourier Transform (definicion)

Parejas en frecuencia y tiempo (o espacio)

Necisitamos un basis de funciones `independiente linearmente' (linear independent)

En caso de Fourier transformes: funciones sin y cosine

e.j. simple - `top hat function' : derivacion de S(f)

$$s(t) = \int_{-\infty}^{+\infty} S(f)\, e^{i2\pi ft}\, df$$

$$S(f) = \int_{-\infty}^{+\infty} s(t)\, e^{-i2\pi ft}\, dt.$$

$$e^{i\phi} = \cos\phi + i\sin\phi$$

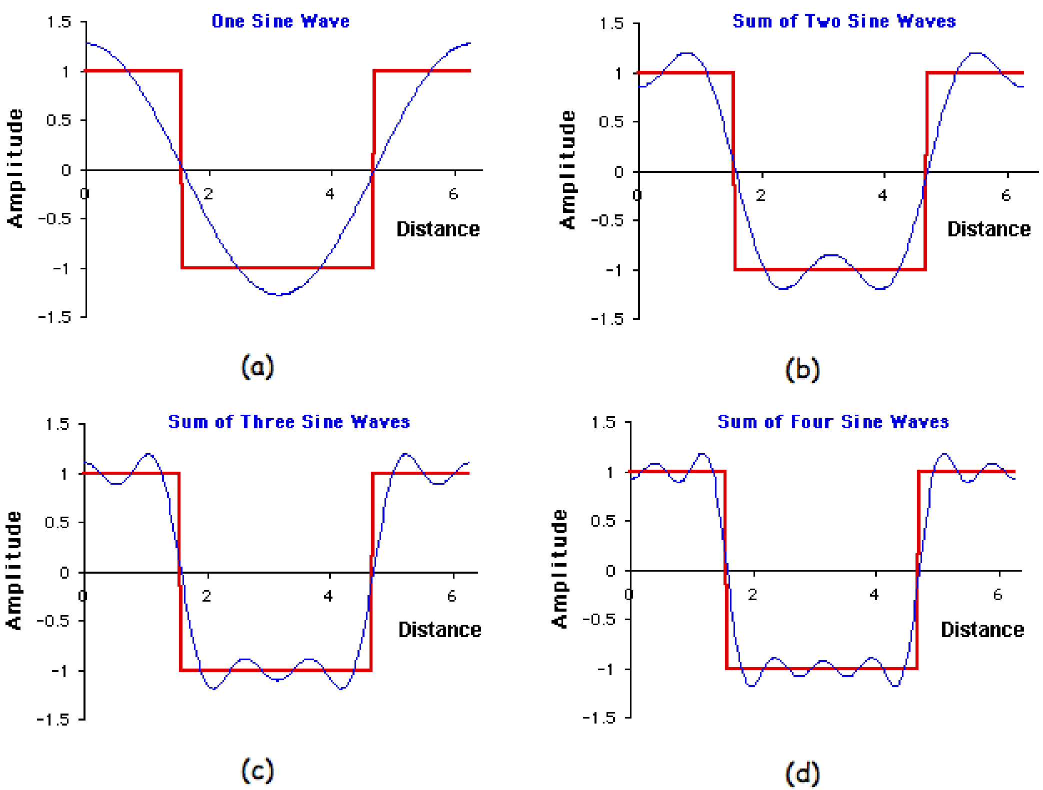
$$e^{i2\pi ft} = \cos 2\pi ft + i\sin 2\pi ft.$$

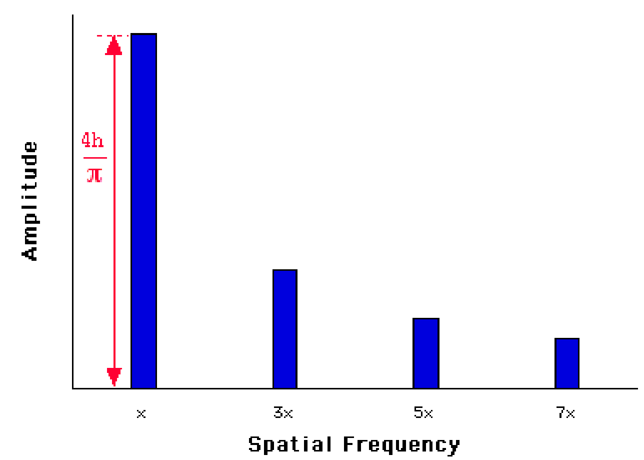
Transforme de Fourier

$$f(x) = \frac{1}{2}a_0 + a_1\cos(x) + a_2\cos(2x) \ldots + a_n\cos(nx)$$
$$+ b_1\sin(x) + b_2\sin(2x) \ldots + b_n\sin(nx)$$

Transform para un 'Square Wave'

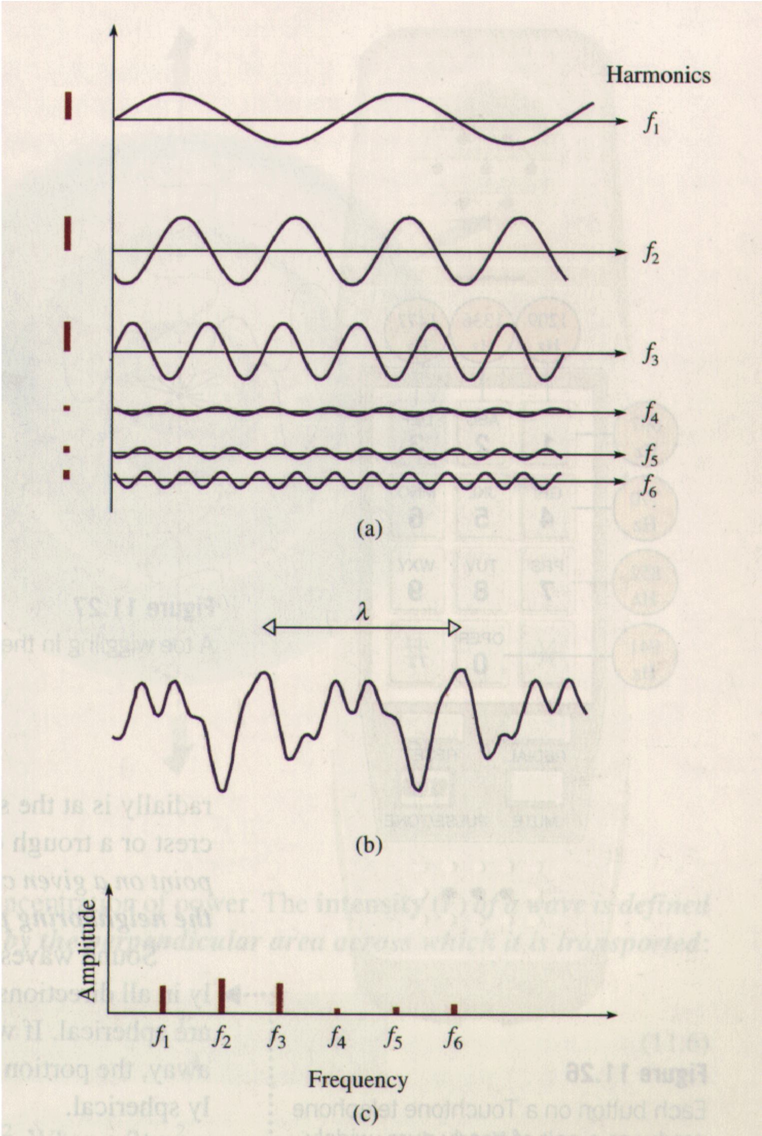
$$f(x) = \frac{4h}{\pi}\left(\sin(x) + \frac{1}{3}\sin(3x) + \frac{1}{5}\sin(5x) + \frac{1}{7}\sin(7x) + \ldots\right)$$



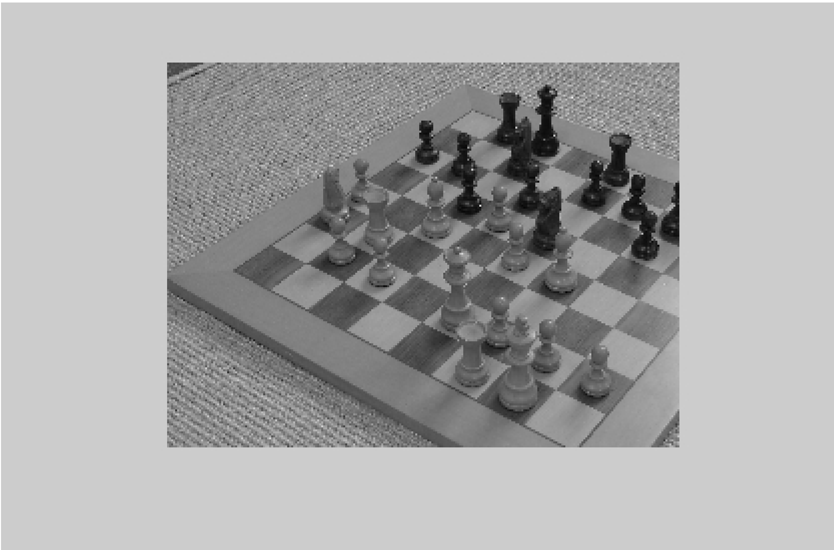


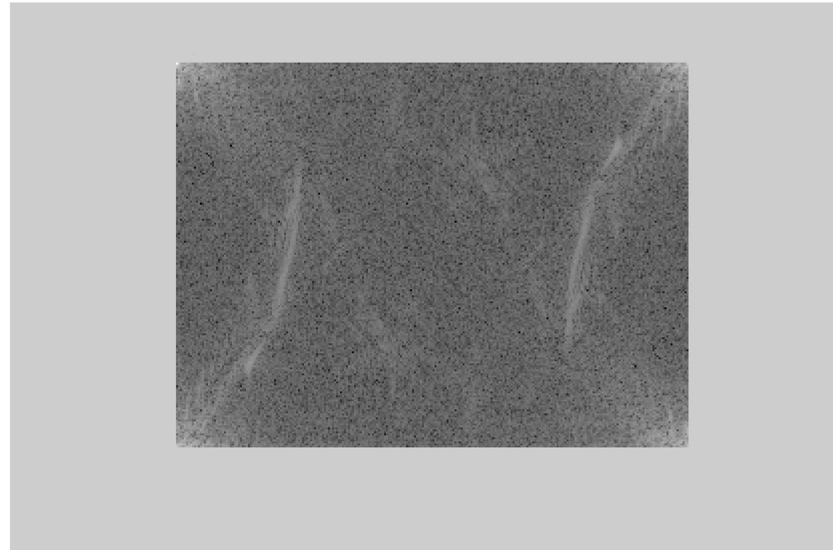
Figures from Wiki-books (author ?)

Transforme de Fourier

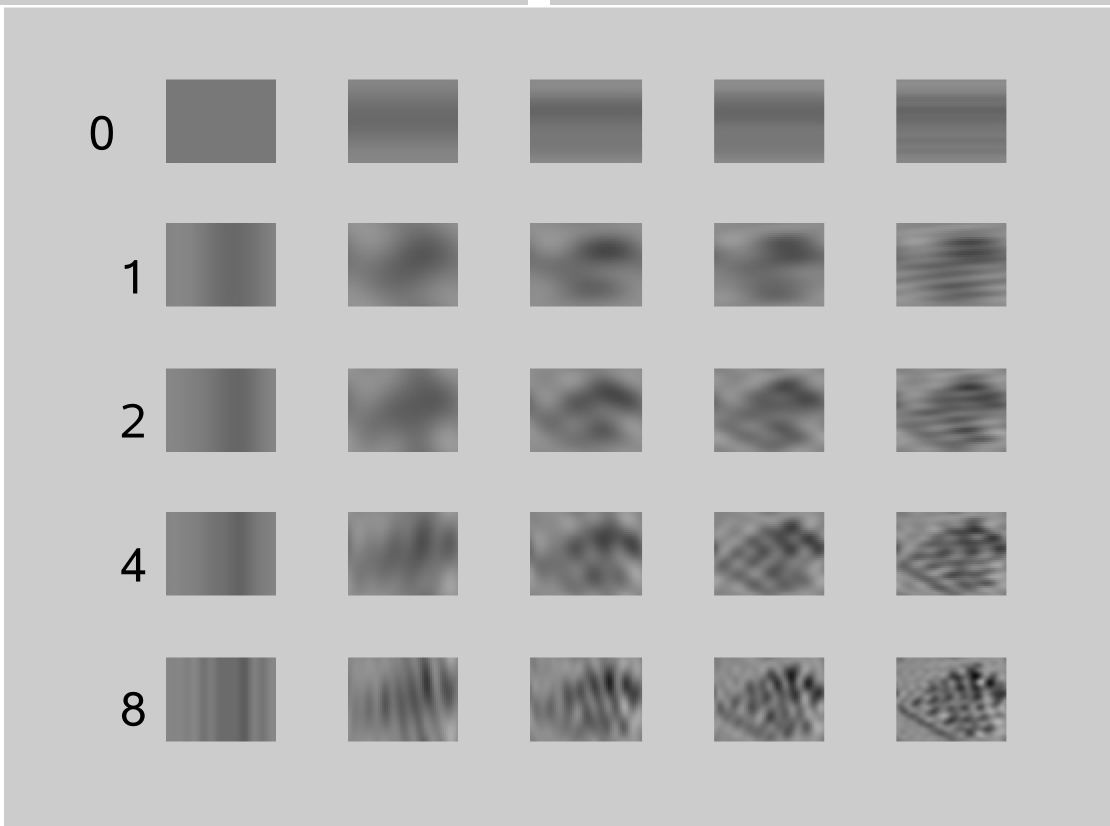


Fourier en dos dimensiones: imagenes

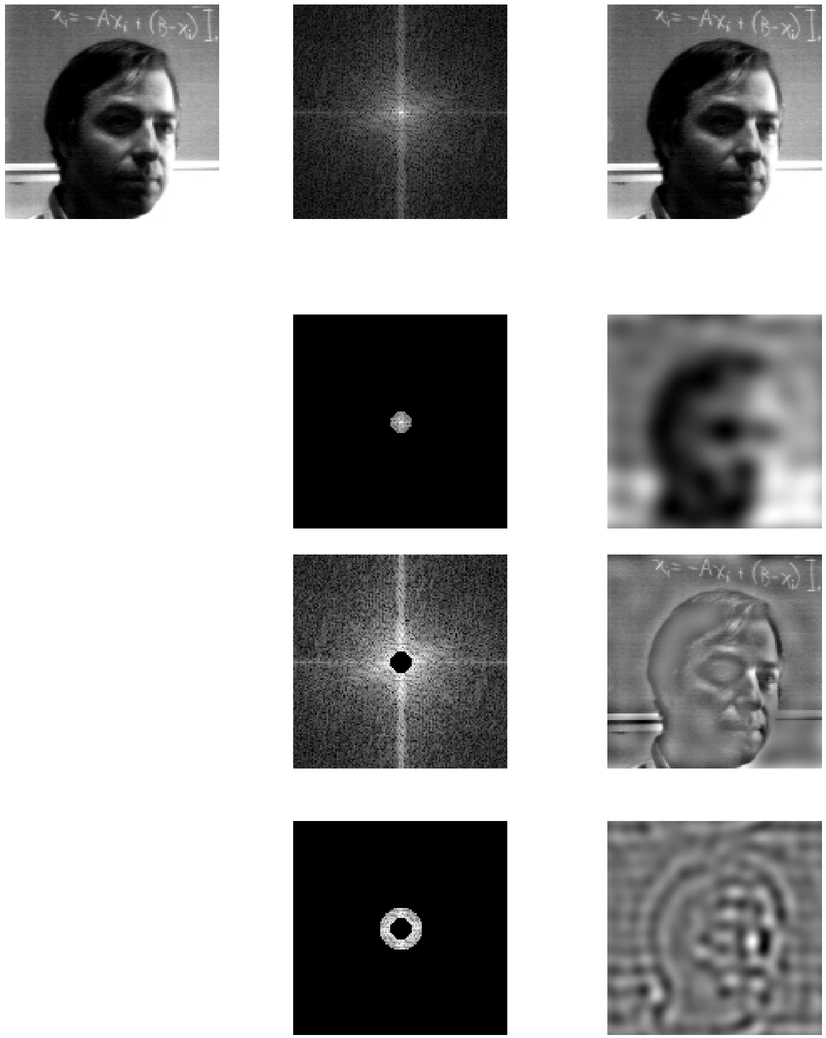




amplitude
y
fase



Sussex
Univ.
webpage

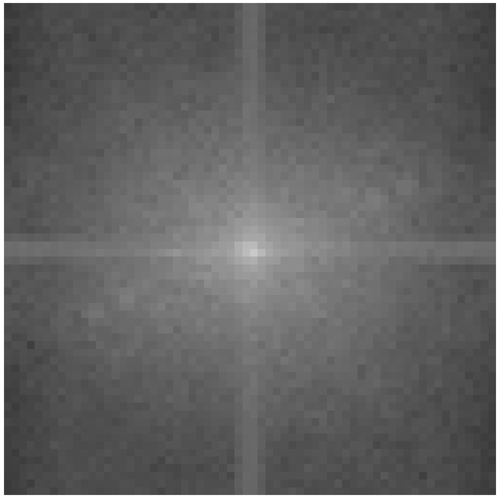


From Steven Lehar's Fourier tutorial webpage

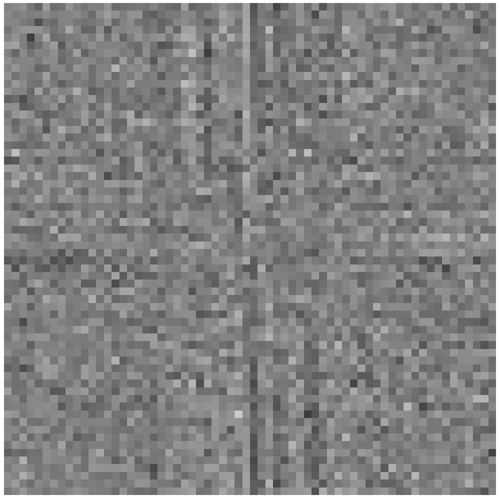
Importancia del fase

Imagen original

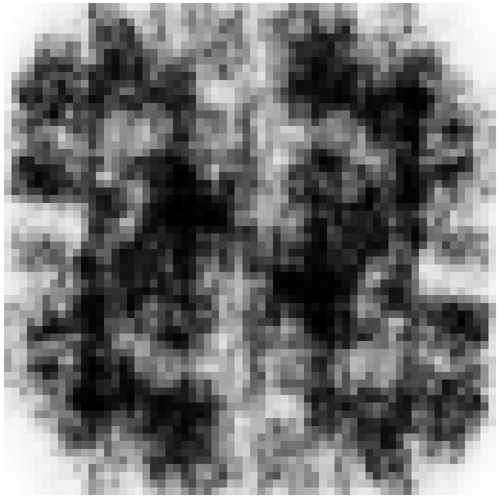
Fourier: log Amplitud



Fourier: fase



from tutorial on
Bob Fisher's homepage
Univ. of Edinburgh

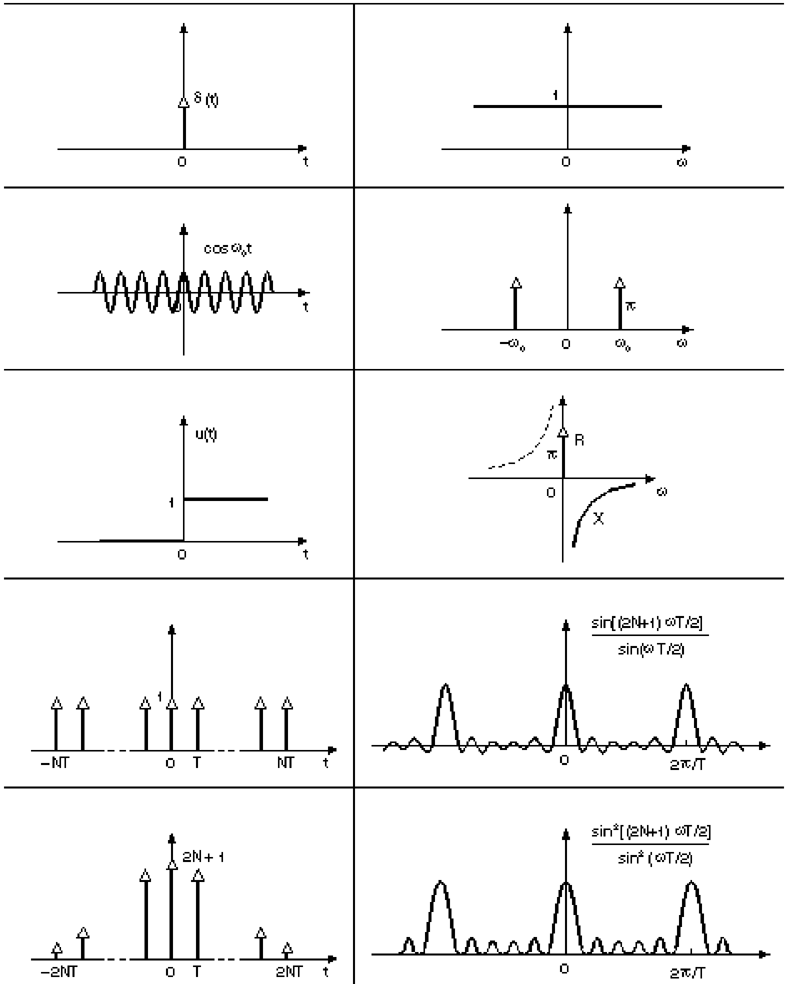


Inverse Fourier
sin fase
informacion

Ejemplos

Transforms of singularity functions

$f(t) \longleftrightarrow F(\omega)$

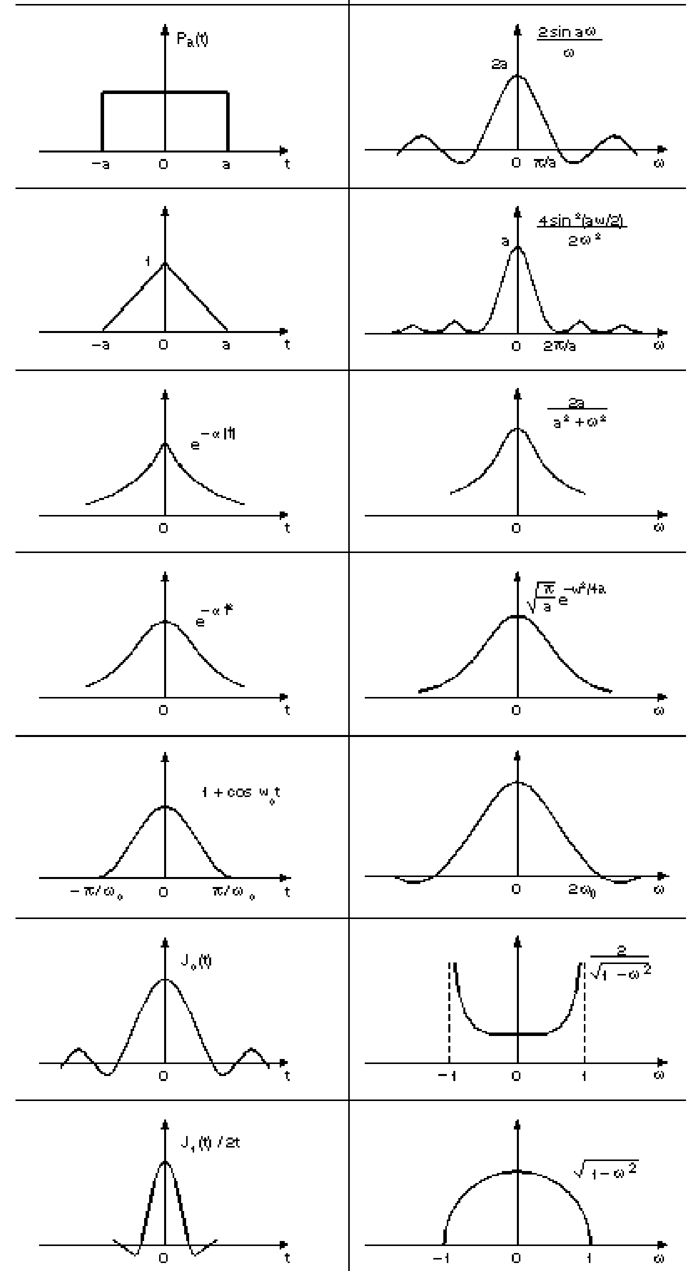


$$\delta(x)\delta(y) \Leftrightarrow 1 \qquad \delta(x) \Leftrightarrow 2\pi\delta(v)$$

$$\varphi(x) \Leftrightarrow 2\pi\Phi(u)\delta(v) \qquad \varphi(x)\delta(y) \Leftrightarrow \Phi(u)$$

Examples of Fourier transforms

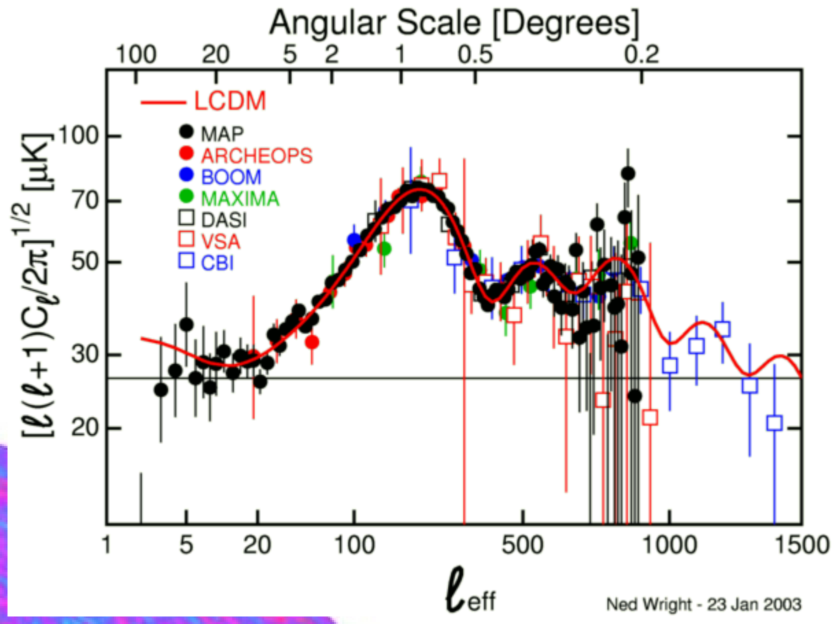
$f(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} F(\omega)e^{j\omega t}d\omega$	$F(t) = \int_{-\infty}^{\infty} f(t)e^{-j\omega t}dt$

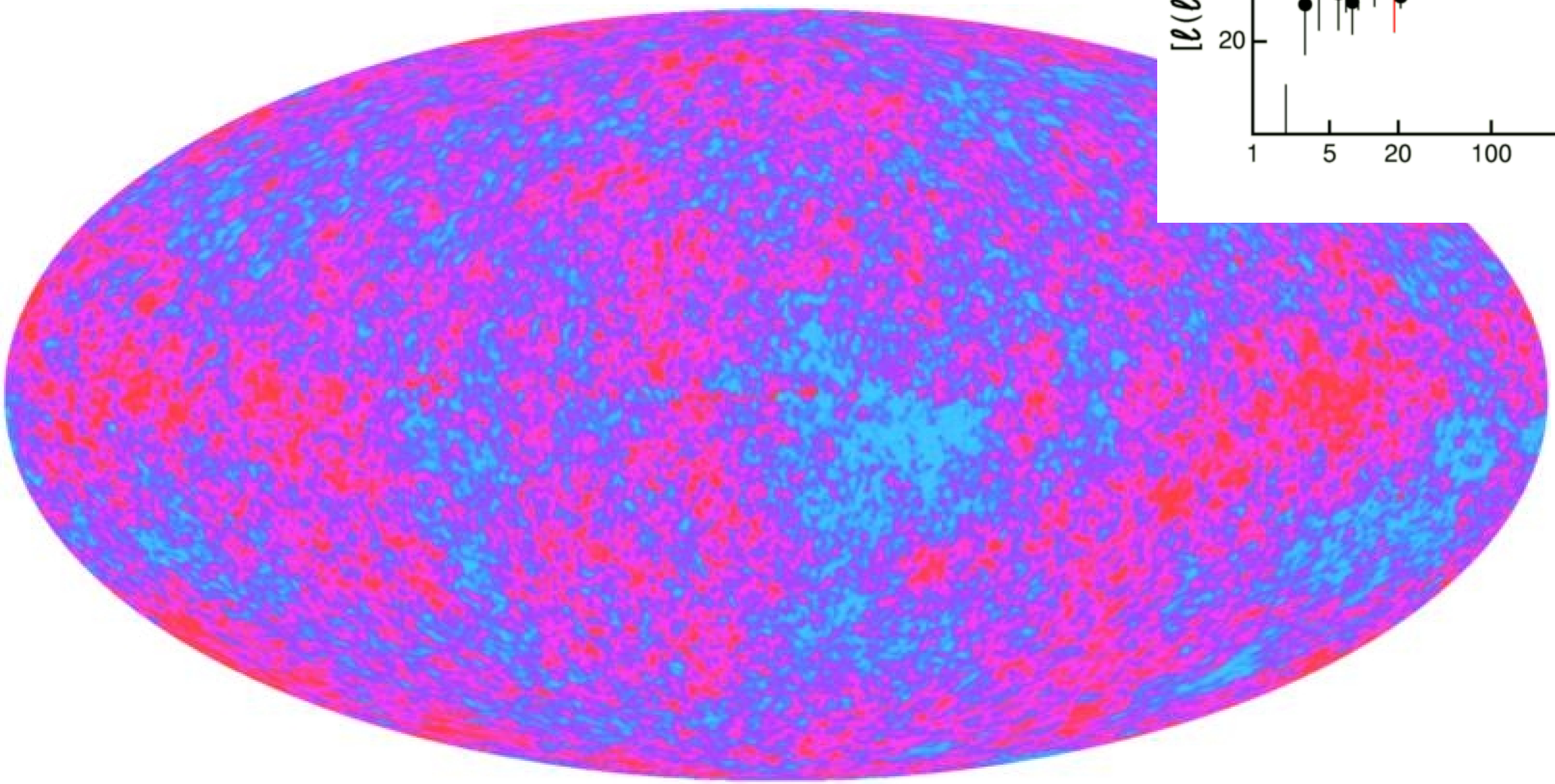


Fourier transform theorems

$f(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} F(\omega)e^{j\omega t}d\omega$	$F(t) = \int_{-\infty}^{\infty} f(t)e^{-j\omega t}dt$
$f(at)$	$\frac{1}{\|a\|}F\left(\frac{\omega}{a}\right)$
$f^*(t)$	$F^*(-\omega)$
$F(t)$	$2\pi f(-\omega)$
$f(t-t_0)$	$F(\omega)e^{-jt_0\omega}$
$f(t)e^{j\omega_0 t}$	$F(\omega-\omega_0)$
$f(t)\cos\omega_0 t$	$\frac{1}{2}[F(\omega+\omega_0)+F(\omega-\omega_0)]$
$f(t)\sin\omega_0 t$	$\frac{1}{2}[F(\omega+\omega_0)-F(\omega-\omega_0)]$
$\frac{d^n f(t)}{dt^n}$	$(jw)^n F(\omega)$
$(-jt)^n f(t)$	$\frac{d^n F(\omega)}{d\omega^n}$
$m_n = \int_{-\infty}^{\infty} t^n f(t)dt$	$F(\omega) = \sum_{n=0}^{\infty}\frac{m_n}{n!}(-j\omega)^n$
$\int_{-\infty}^{\infty} f_1(\tau)f_2(t-\tau)d\tau$	$F_1(\omega)F_2(\omega)$
$\int_{-\infty}^{\infty} f(t+\tau)f^*(\tau)d\tau$	$\|F(\omega)\|^2$
$\int_{-\infty}^{\infty} \|f(t)\|^2 dt = \frac{1}{2\pi}\int_{-\infty}^{\infty}\|F(\omega)\|^2 d\omega$	
$f(t)+j\hat{f}(t)$	$2F(\omega)U(\omega)$
$\hat{f}(t)$	$-j\ \mathrm{sgn}\ \omega F(\omega)$
$\sum_{n=-\infty}^{\infty} f(t+nT) = \frac{1}{T}\sum_{n=-\infty}^{\infty} F\left(\frac{2\pi n}{T}\right)e^{j2\pi nt/T}$	

WMAP: fluctuaciones en la temperatura del CMB
rango: 0.0005 K desde frio (azul) a caliente (rojo)





Fourier transforms are for square images: we have the surface of a sphere.
So use Spherical Harmonics instead.
Analogous to the Fourier power spectrum is the angular power spectrum

WMAP: fluctuaciones en la temperatura del CMB
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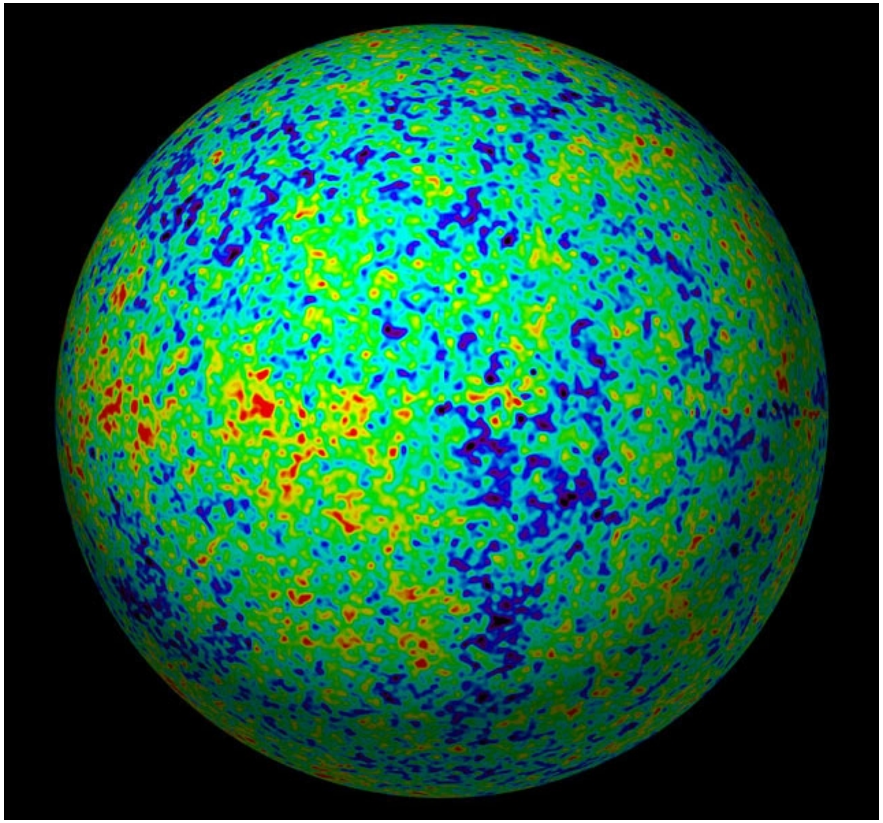

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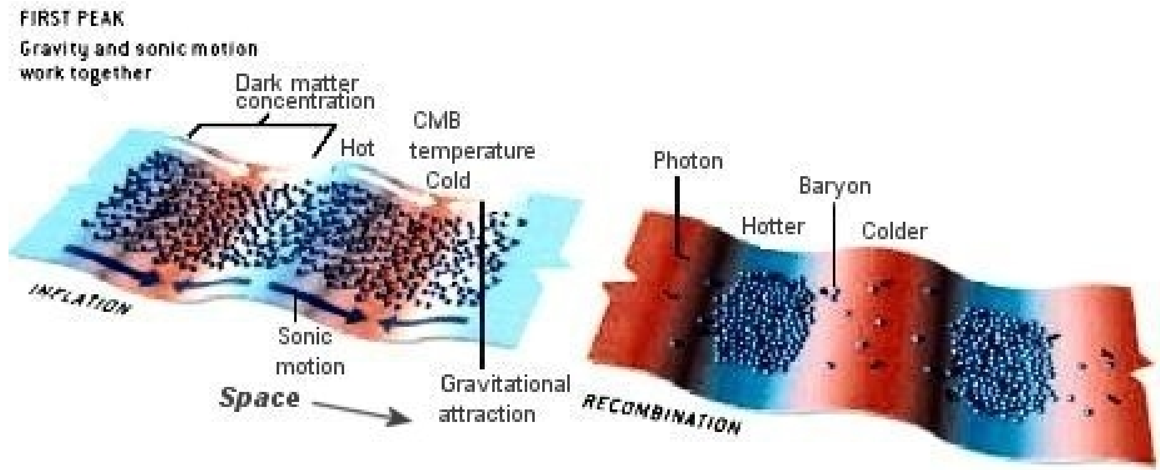
Analogous to the Fourier power spectrum is the angular power spectrum

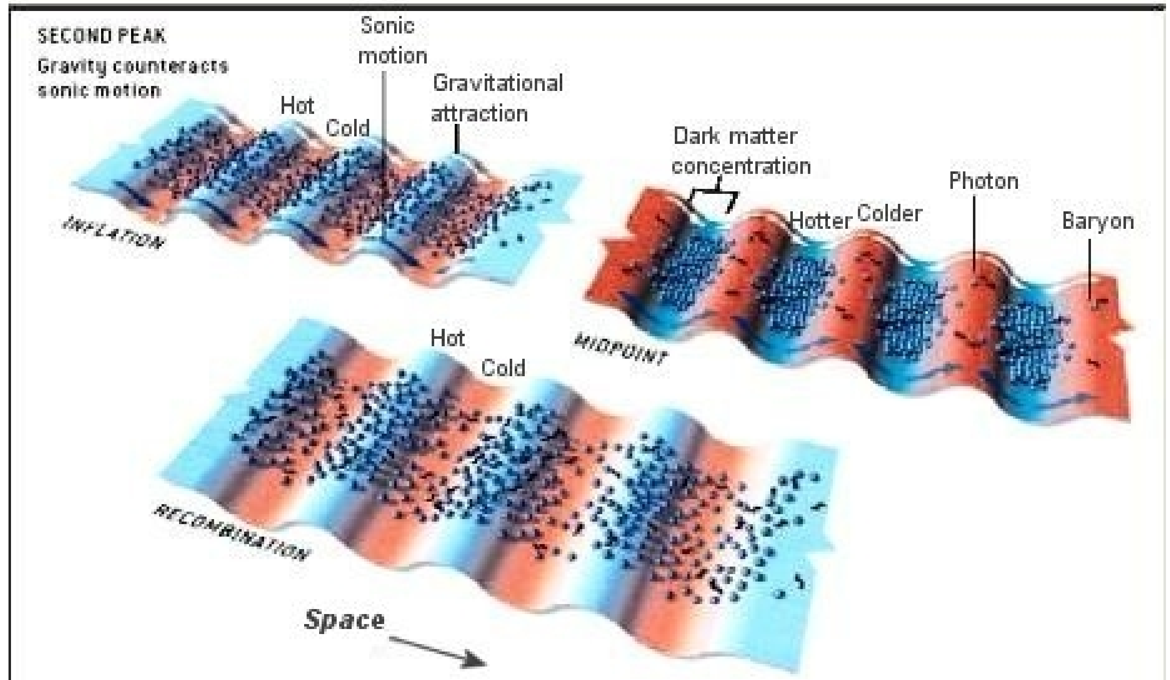
$$\frac{\Delta T(\theta,\phi)}{T} = \sum_{\ell m} a_{\ell m} Y_{\ell m}(\theta,\phi)$$



Fourier:

$$F(x) = \Sigma_k \{G_k \cos(kx)\} \text{ ---------- } (1)$$

$k = 2\pi/L$, L is the wavelength.



Spherical Harmonics

$$F(\theta,\phi) = \Sigma_l \Sigma_m a_{lm} Y_{lm}(\theta,\phi) \text{ ---------- } (2)$$

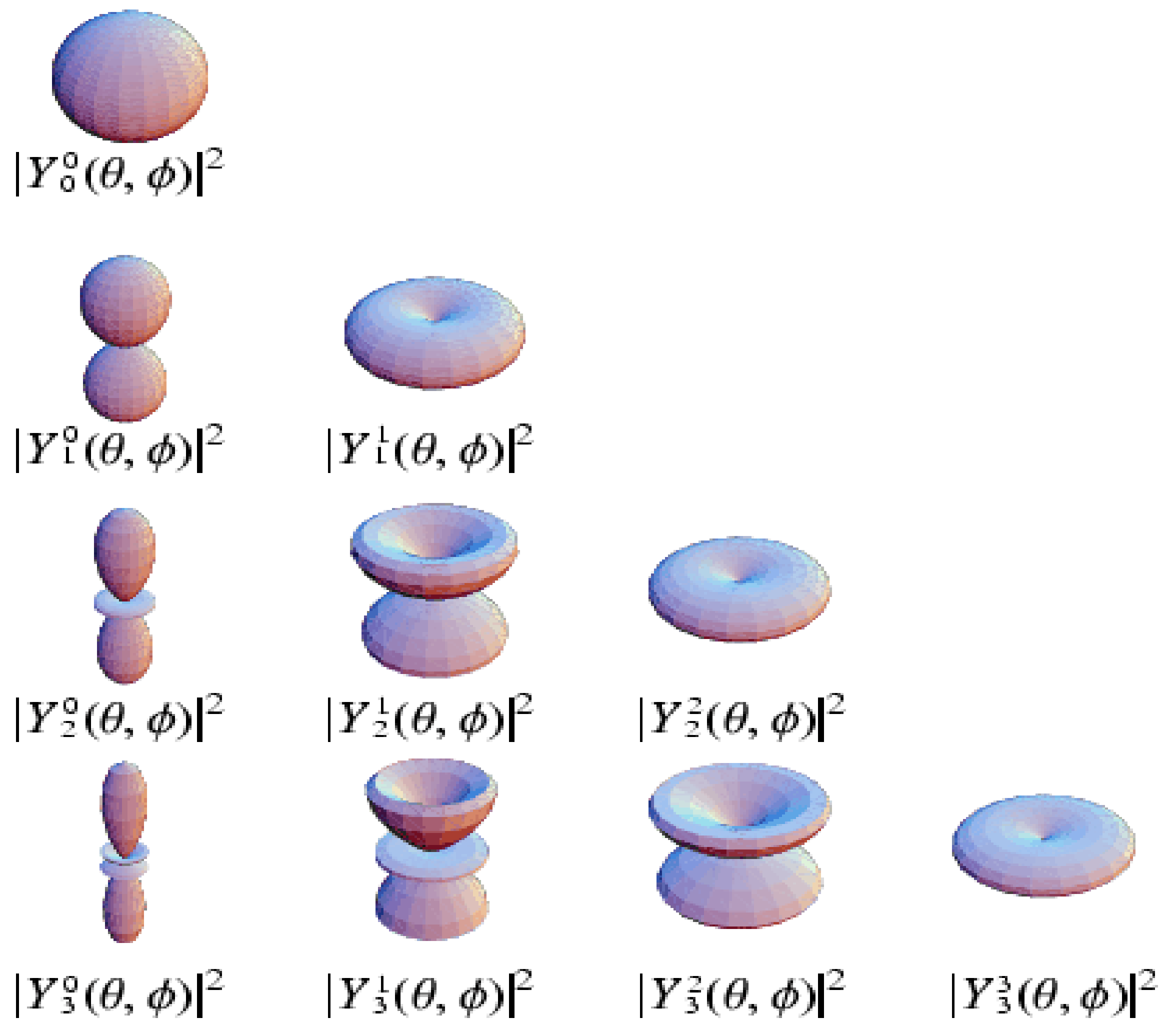
$Y_{lm}(\theta,\phi)$, where l=0 denotes the monopole, l=1 the dipole, l=2 the quadrupole, ..., and m

integer between $-l$ and l.

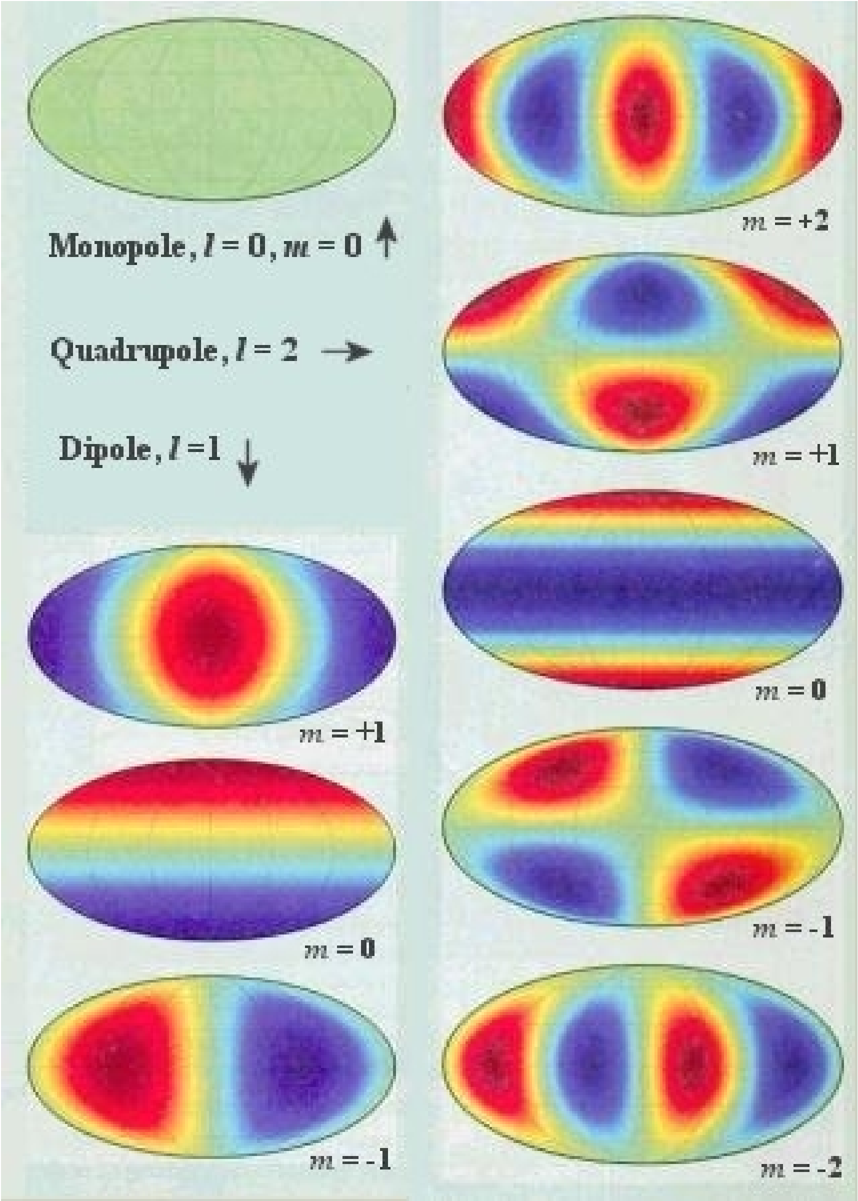
Normally plot $l(l+1)\, C_l$ versus l, where $C_l = \Sigma_m |a_{lm}|^2$. because it is approx equal to the log power per unit l

$$(\Delta T/T)_\theta \approx \sqrt{\ell(\ell+1)C_\ell/2\pi}$$

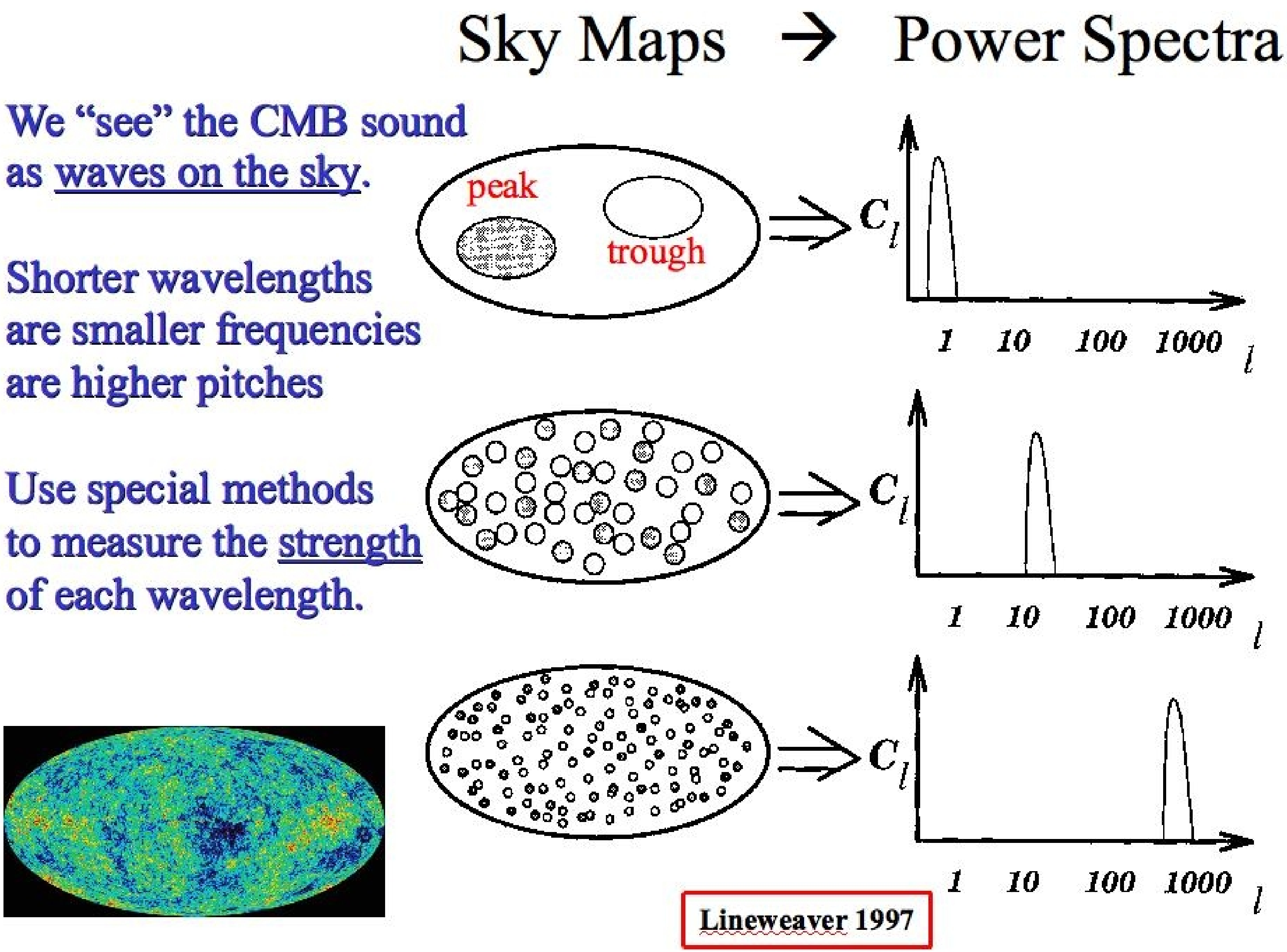
Spherical Harmonics



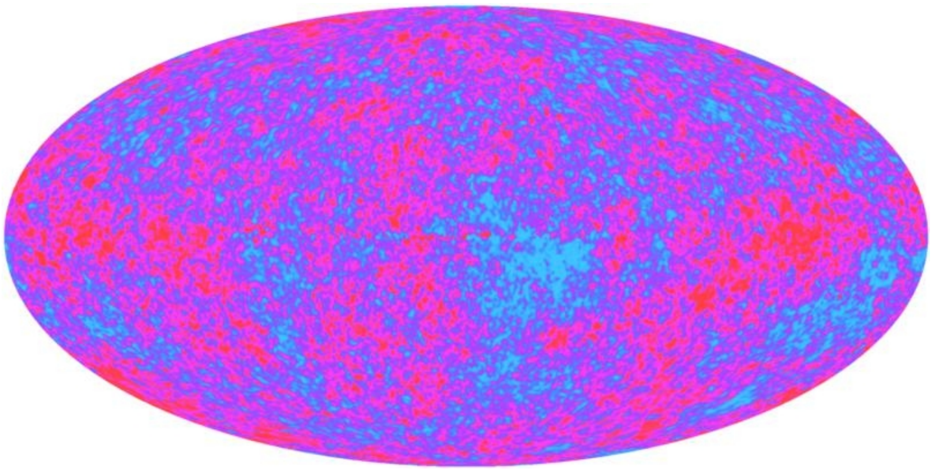
2 Dimensions (surface of a sphere)

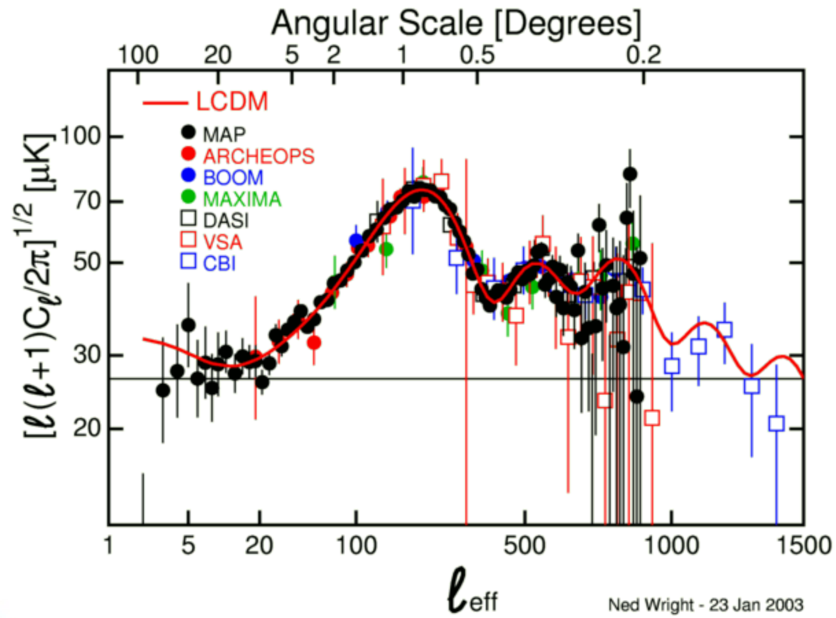


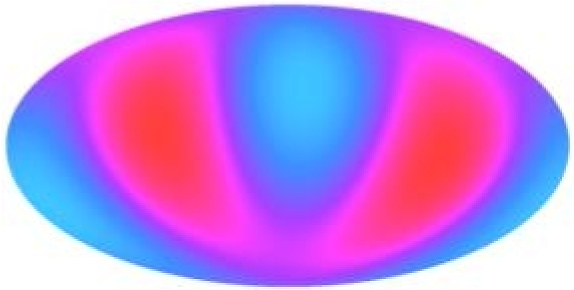
Ejemplos


Sky Maps → Power Spectra
We "see" the CMB sound
as waves on the sky.
Shorter wavelengths
are smaller frequencies
are higher pitches
Use special methods
to measure the strength
of each wavelength.
peak
trough
C_l
1 10 100 1000 l
Lineweaver 1997

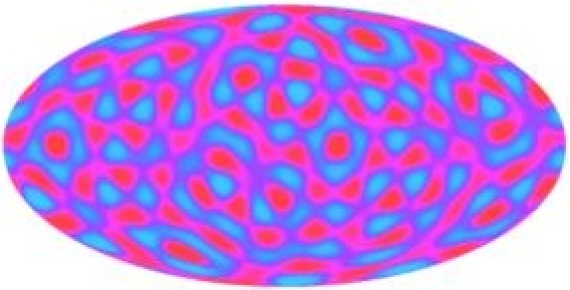
CMB: Power Spectrum: 1 deg on sky -> l = 180



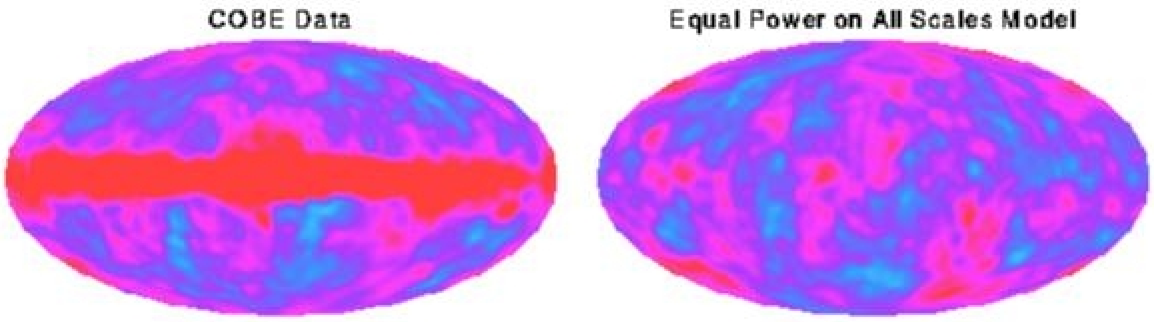




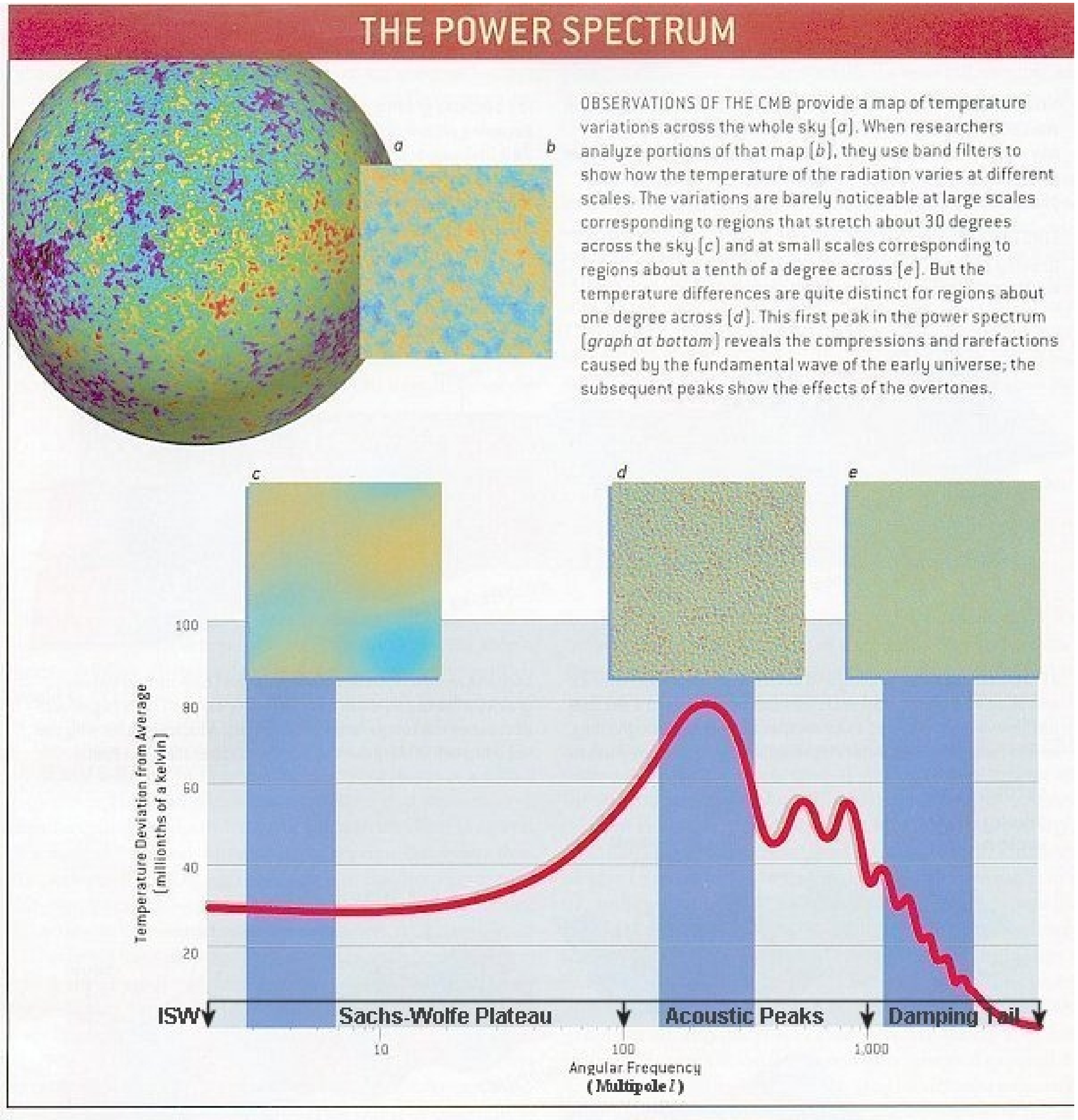
$\ell = 2$

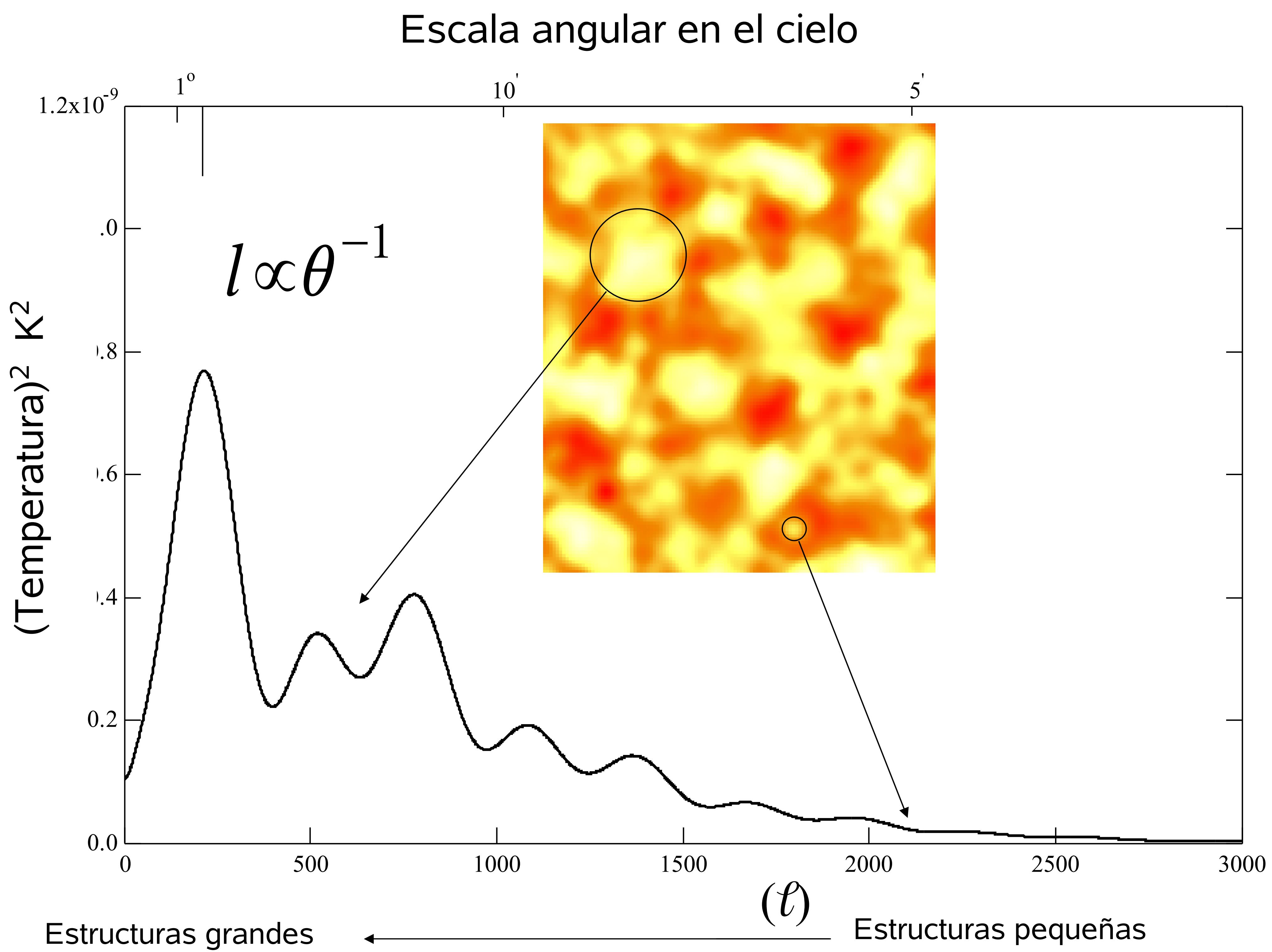


$\ell = 16$



Ejemplos

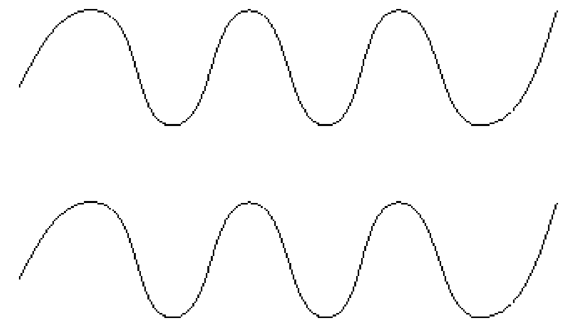

THE POWER SPECTRUM
OBSERVATIONS OF THE CMB provide a map of temperature variations across the whole sky (a). When researchers analyze portions of that map (b), they use band filters to show how the temperature of the radiation varies at different scales. The variations are barely noticeable at large scales corresponding to regions that stretch about 30 degrees across the sky (c) and at small scales corresponding to regions about a tenth of a degree across (e). But the temperature differences are quite distinct for regions about one degree across (d). This first peak in the power spectrum (graph at bottom) reveals the compressions and rarefactions caused by the fundamental wave of the early universe; the subsequent peaks show the effects of the overtones.
a
b
c
d
e
Temperature Deviation from Average (millionths of a kelvin)
100
80
60
40
20
ISW
Sachs-Wolfe Plateau
Acoustic Peaks
Damping Tail
10
100
1,000
Angular Frequency
(Multipole l)


Escala angular en el cielo
1°
10'
5'
1.2x10-9
.0
.8
.6
.4
0.2
0.0
(Temperatura)² K²
$l \propto \theta^{-1}$
0
500
1000
1500
2000
2500
3000
(ℓ)
Estructuras grandes
Estructuras pequeñas

Interferometria y Interferencia

Interferencia constructiva

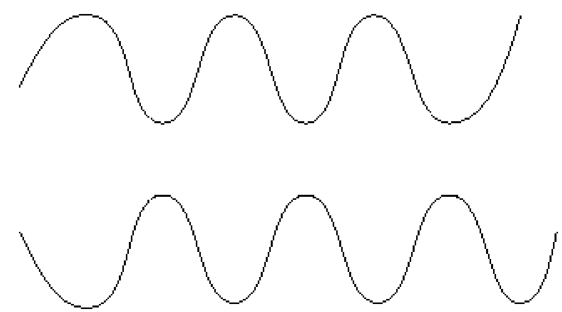
(amplitudo netto = max)



$A = A_0 \cos(\omega t + \phi)$

Interferencia destructiva

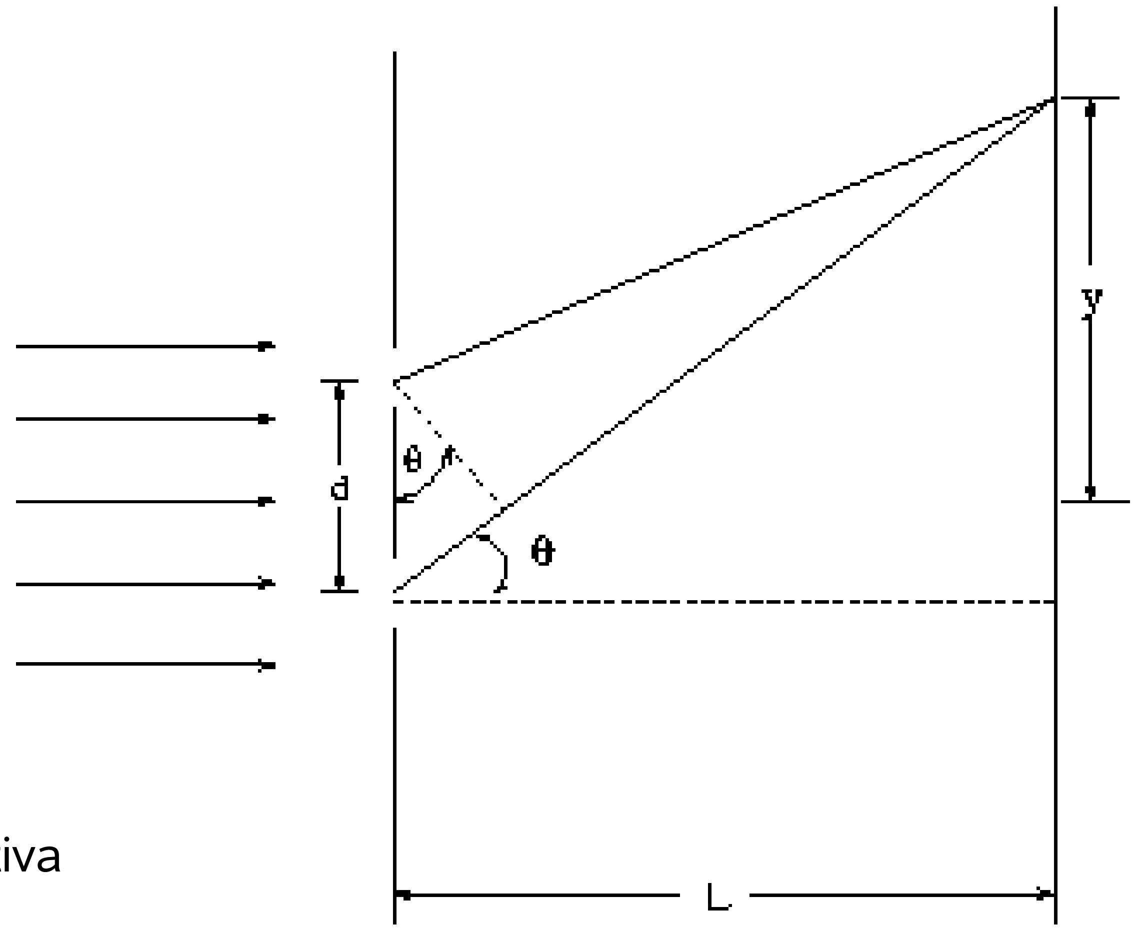
(amplitudo netto = 0)



Dos ondas comenzar en fase en el mismo punto y viajan distancia r1 y r2, resp.

$r1 - r1 = m\lambda$ (m = 1,2,3....) --> Interferencia constructiva
$r1 - r2 = (m+½)\lambda$ (m = 1,2,3...) --> Interferencia destructiva

Young's Double Slit Experiment



dos rayos, dos “slits”
(tamano del “slits” < λ del luz)

si $d \ll L$, $r1 - r2 = d \sin \Theta$

entonces, quando:

$d \sin \Theta = m \lambda$ (m= +/- 1,2,3...)
interferencia constructiva

$d \sin \Theta = (m+½) \lambda$ (m = +/- 1,2,3...)
interferencia destructiva

La pantalla va a tener bandas brillos y bandas oscuos alternadas:

se llama : Interference fringes (y con un foton?)

si d << L, r1 – r2 = d sin Θ
si y << L, sin Θ = y / L

entonces quando:

d sin Θ = m λ   (m= ±1,±2,±3...)
interferencia constructiva

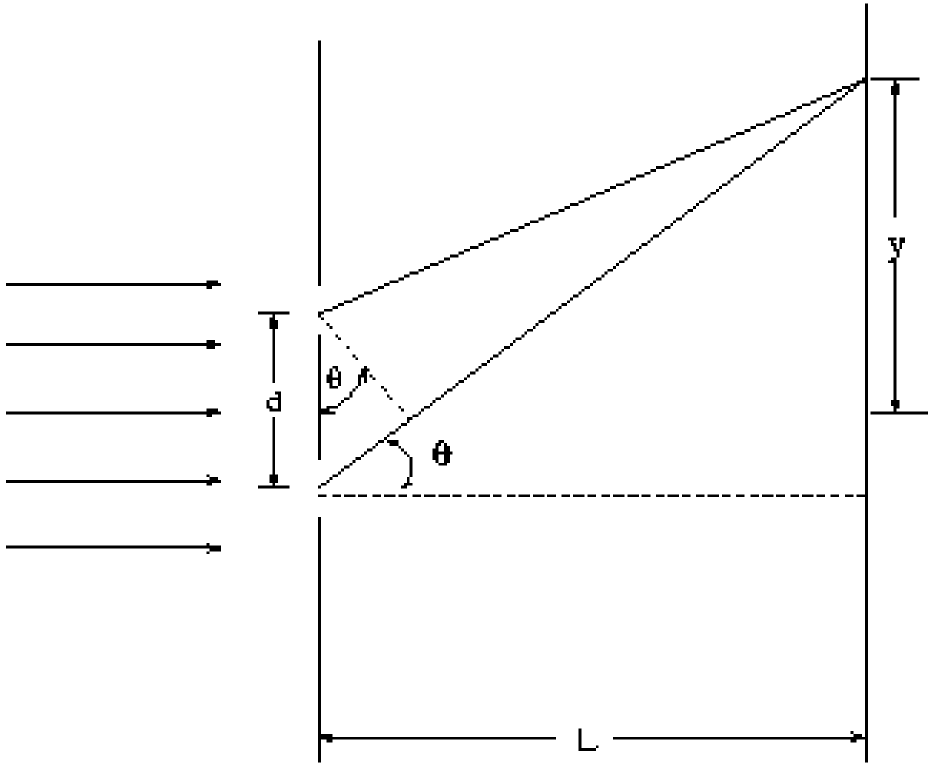
y = (m λ L) / d   (banda brillo)

d sinΘ = (m+½) λ   (m = ±1,±2,±3...)
interferencia destructiva

y = (m +½) λ L / d   (bando oscuro)

distancia entre bandos brillos (o oscuros)

Δy = λ L / d

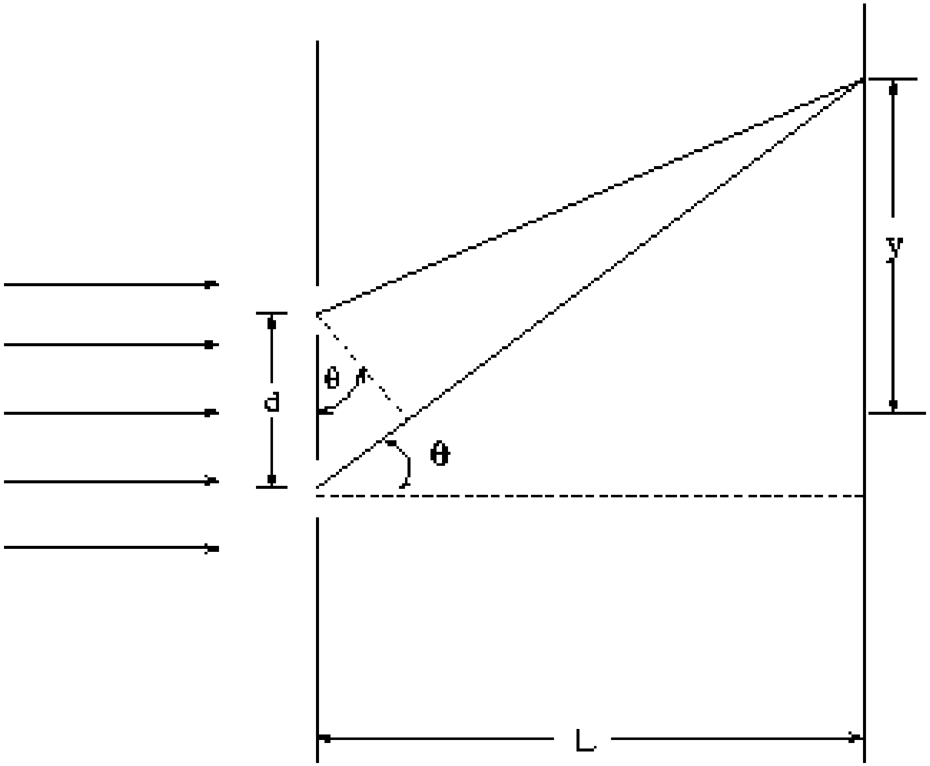


Ver JAVA applet sobre Interferencia.

Imagen en la pantalla es una componente del Transform Fourier del Fuente

slits -> telescopios (radio o optico)

pantalla -> correlador
    optico : light delay (espejos)
        direct interference
        fringe image
        medir
    radio : delay (cables/fibros)
        correlador
        medir amplitude y fase



Various ejemplos:
    fuentes puntuales o extendido
    posicion del fuente y fase
    "d" pequena o grande

Necisitamos various "d"'s
--> Earth Rotation Synthesis

recordar: $\Delta y = \lambda L / d$
No ocupamos "L" in radio interferometria
facil si "d" es in unidades de λ
-->> el plano *uv*

Equatorial: Earth

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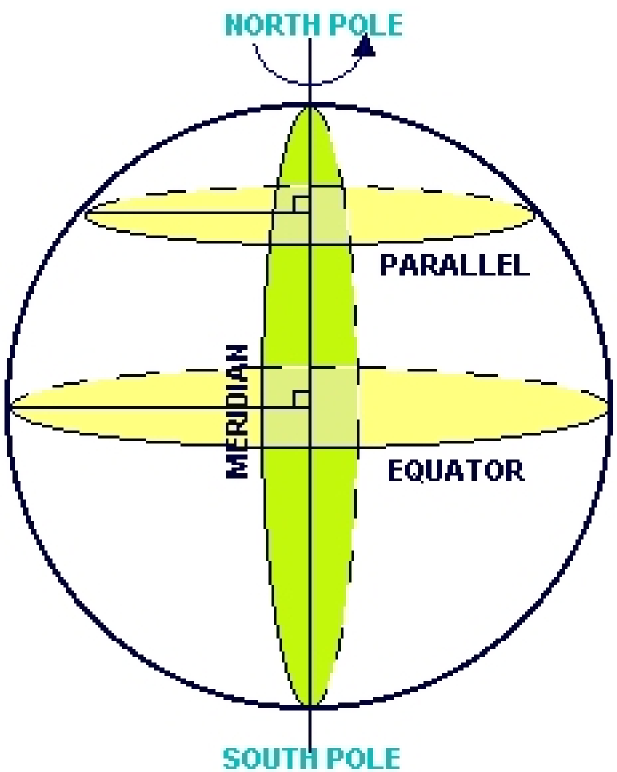
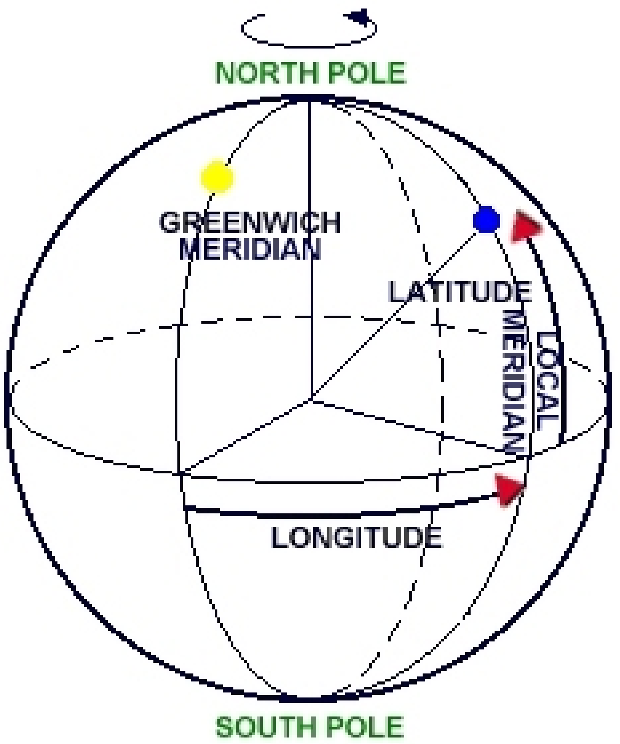
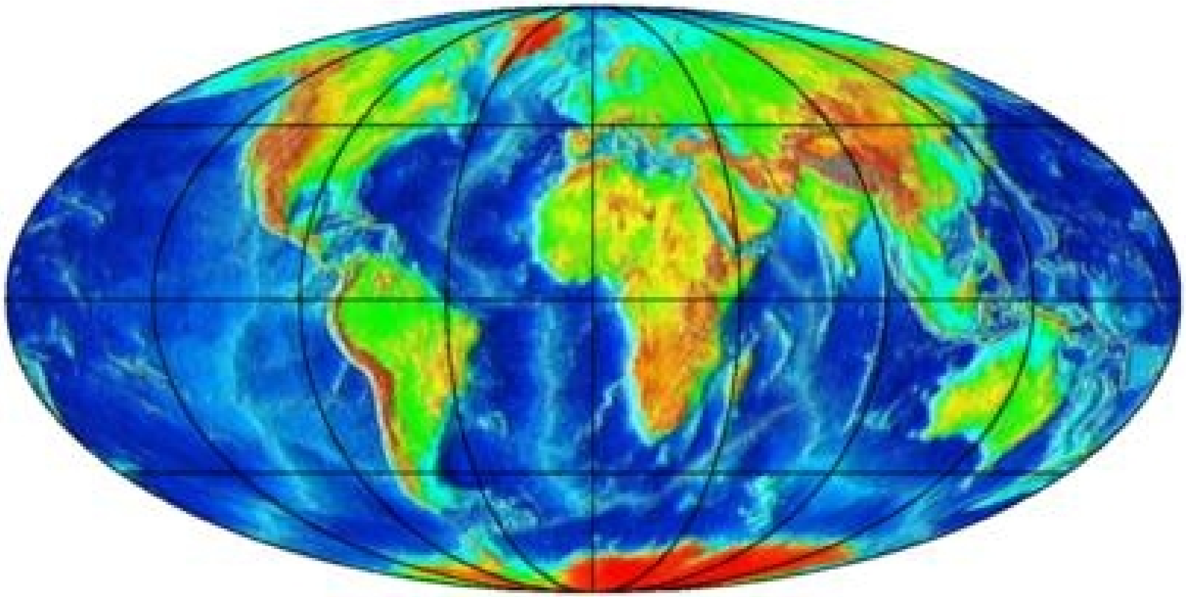
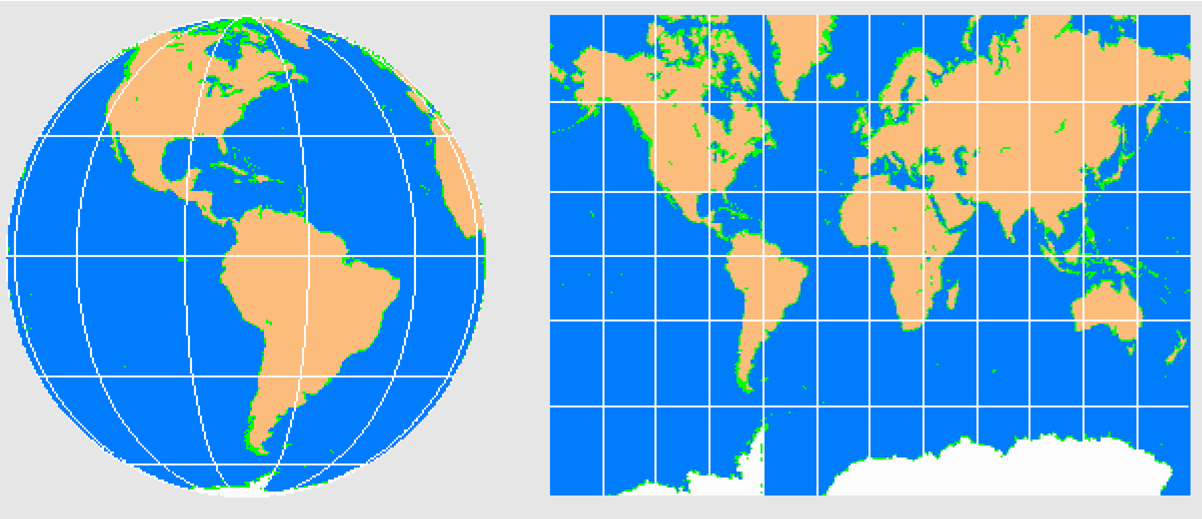




Equal area projection

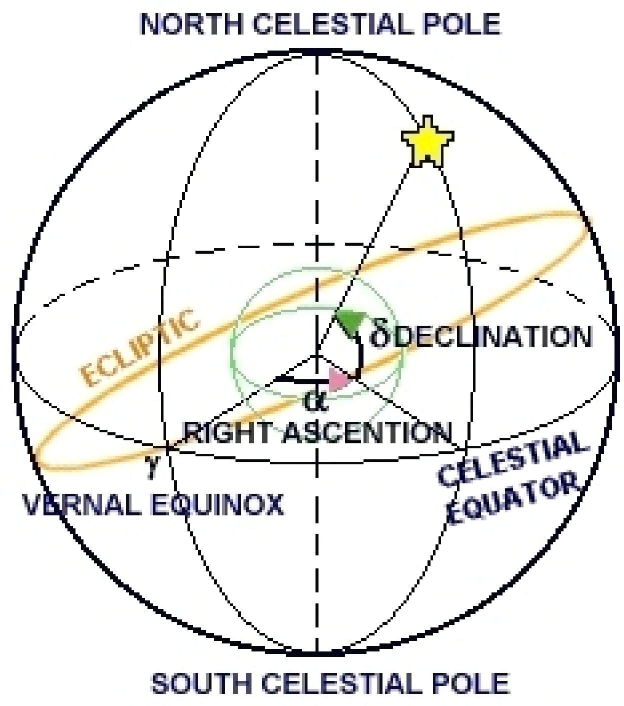
Equatorial: Earth (Aitoff Projection)





Equatorial: Sky (earth centered)

Earth vs. Sky: difference in E-W directions



Azumuth & Altitude: depends on location of observer & time

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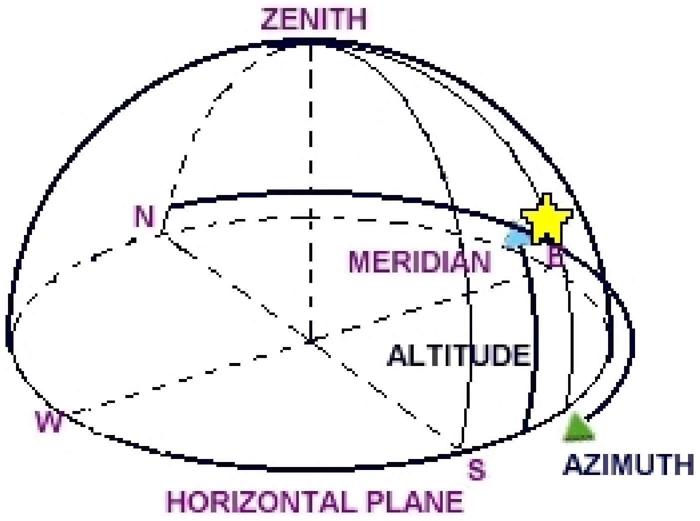
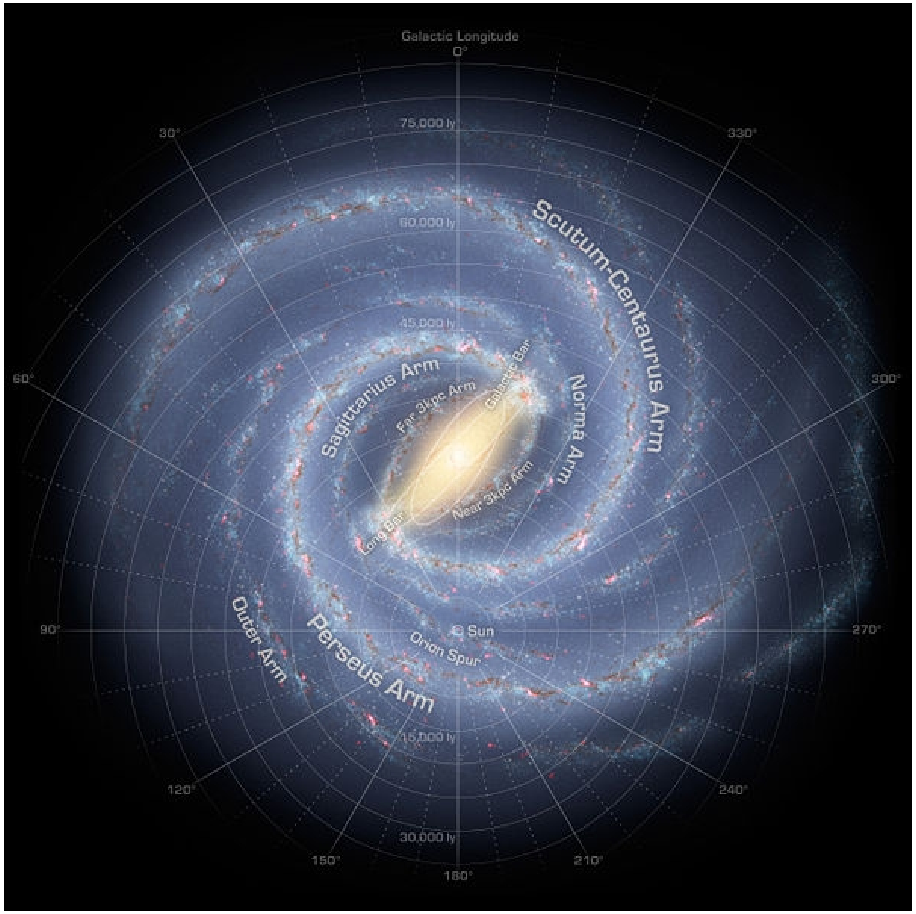
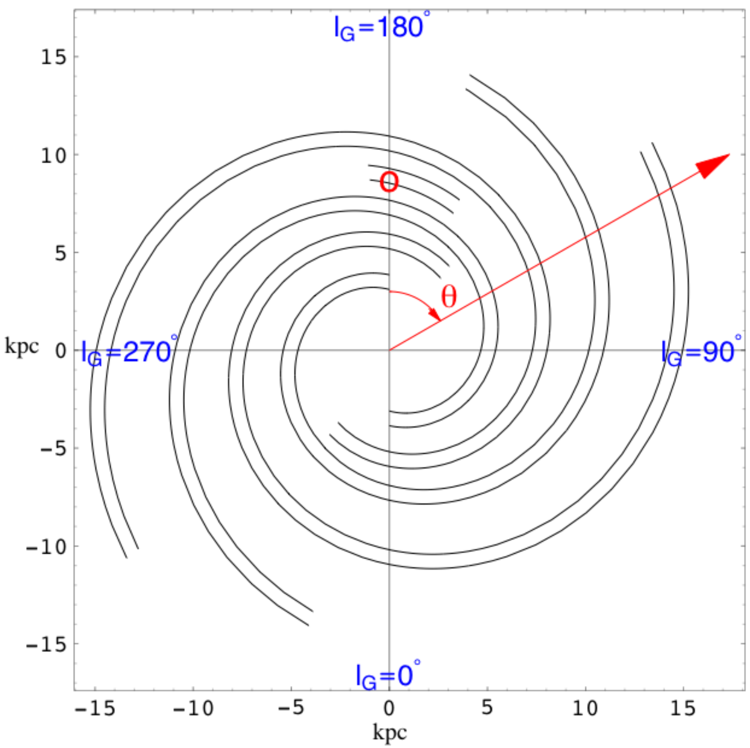


Galactic Coordinates: l,b

desde NGP: Galactic l incrementa en sentido anti-horario (l=0 es el GC)





Super-Galactic Coordinates:

Aitoff Projection (equi-distant? o equi-area?)

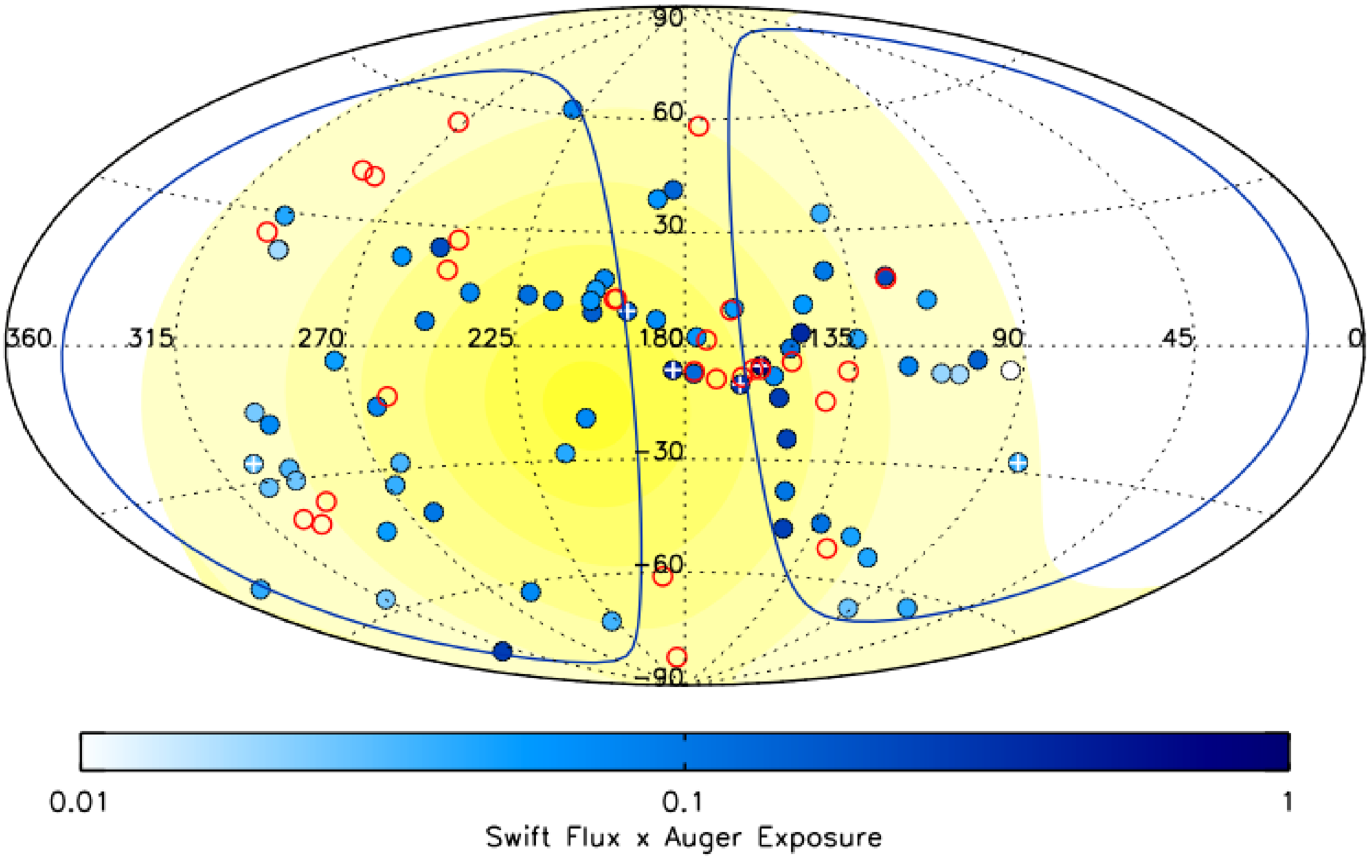


Figure 1. Map of Auger UHECRs (open red circles) and BAT AGN within 100 Mpc (filled blue circles) in supergalactic coordinates (de Vaucouleurs et al. 1976). The blue colour depth is scaled by the hard X-ray flux and Auger exposure, relative to Cen A. The 6 AGN in the catalogue within 20 Mpc are marked with white crosses, with Cen A at $(159.7°, -5.2°)$. Yellow contours have equal integrated exposures. Blue boundaries show where the AGN catalgoue is incompete due to the Galactic plane, $|b| < 15°$.

Galactic Coordinates: l,b (Aitoff Projection)

equal area projection
Note Supergalactic plane (dashed line). Tambien "zone of avoidance"