# Cosmology Lectures

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# Lecture Outline

- THEME: A simple cosmological model, a vacuum energy dominated flat universe with scale invariant Gaussian random phase fluctuations, fits a host of astronomical observations. What is the basis of this model? What are the key open questions?

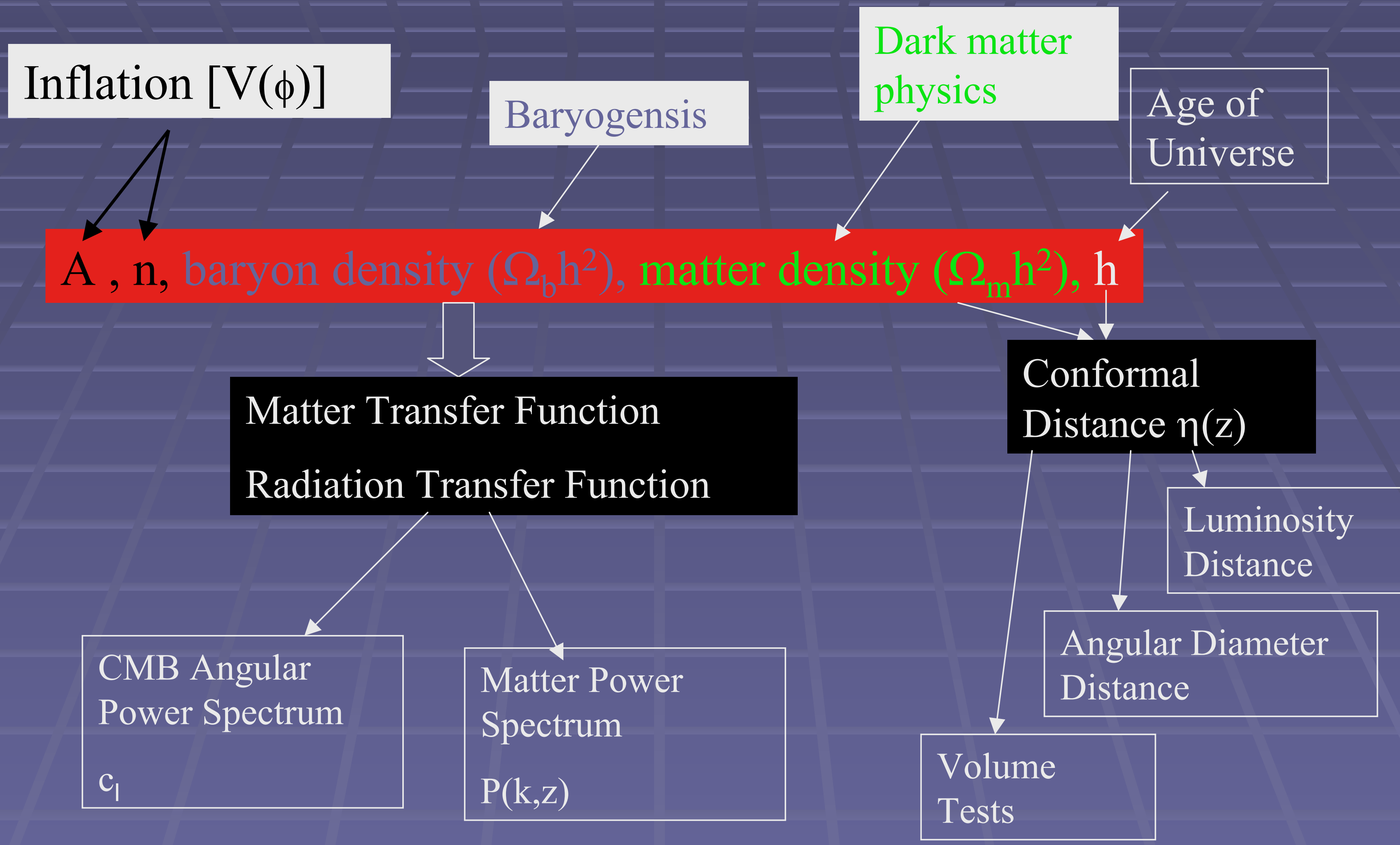
I. General Relativity: Dynamics and Linear Theory

II. Inflation: Motivation and Implications

III. Microwave Background Fluctuations

IV. Open Questions and Future Observations

# Minimal Cosmological Model



# General Relativity: A Review

- Space is not absolute.  Only relative positions are meaningful.
- Matter tells space how to curve
- Curvature of space tells matter how to move: particles move on “straight lines”

# Metric Theory

$$
\begin{aligned}
(ds)^2 &= (dt)^2 - [(dx)^2 + (dy)^2 + (dz)^2] \\
&= g_{\mu\nu} dx^\mu dx^\nu
\end{aligned}
$$

$$x^\mu = (ct, x, y, z)$$

$$g_{\mu\nu} = \begin{pmatrix} -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

# General Relativity: A Review

- Space is not absolute.  Only relative positions are meaningful.
- Matter tells space how to curve
- Curvature of space tells matter how to move: particles move on “straight lines”

# Particles move on straight lines

$$\delta \int g_{\mu\nu} u^\mu u^\nu = 0$$

$$\ddot{x}^\mu + \Gamma^\mu_{\alpha\beta} \dot{x}^\alpha \dot{x}^\beta = 0$$

$$\Gamma^\alpha_{\lambda\mu} = \frac{1}{2} g^{\alpha\nu} \left( \frac{\partial g_{\mu\nu}}{\partial x^\lambda} + \frac{\partial g_{\lambda\nu}}{\partial x^\mu} + \frac{\partial g_{\mu\nu}}{\partial x^\lambda} \right)$$

NEWTONIAN LIMIT:

$$g_{00} = - \left( 1 + \frac{\phi}{c^2} \right)$$

$$\ddot{x} = - \nabla \phi$$

# Matter Distribution determines curvature of space

Riemann Tensor: Unique combination of second derivatives of metric

$$R^{\mu}_{\alpha\beta\gamma}$$

Ricci tensor $R_{\alpha\beta} = R^{\mu}_{\alpha\beta\mu}$ Curvature Scalar $R = R^{\alpha}_{\alpha}$

$$G^{\mu\nu} = R^{\mu\nu} - \frac{1}{2} g^{\mu\nu} R = -\left(\frac{8\pi G}{c^4}\right) T^{\mu\nu}$$

Einstein Equation

$$\nabla^2 \phi = 4\pi G \rho$$

Newtonian limit of Einstein equation

# GR from Least Action Principle

Least Action:

$$S = \int d^4x \sqrt{-g}(R + \Lambda) + \int d^4x \sqrt{-g}\mathcal{L}_{\rm matter}$$

$$g = g^{\alpha}_{\alpha}$$

What is this doing here?

Once you start adding terms, there may be no stopping:

$$S = \int d^4x \sqrt{-g}\left[R + f(\phi)g(R, R_{\mu\nu}R^{\mu\nu}, R_{\alpha\beta\gamma\delta}R^{\alpha\beta\gamma\delta}) + \int d^4x \sqrt{-g}\mathcal{L}_{\rm matter}\right]$$

e.g., Carroll et al., astro-ph/0413001

# Big Bang Cosmology

Homogeneous, isotropic universe ⟺

$$T_{\mu\nu} = \begin{pmatrix} \rho & 0 & 0 & 0 \\ 0 & p & 0 & 0 \\ 0 & 0 & p & 0 \\ 0 & 0 & 0 & p \end{pmatrix}$$

$$(ds)^2 = (dt)^2 - a^2(t)[(dx)^2 + (dy)^2 + (dz)^2]$$

(flat universe)

$$H^2 = \left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3}\rho$$

Freidmann Equation

$$\Omega = \frac{8\pi G\rho}{3H^2} = 1$$

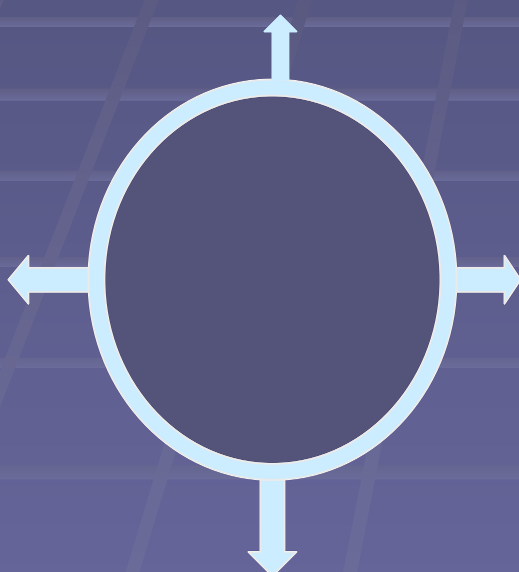
# Pseudo-Newtonian Cosmology

Differentiate Friedmann equation:

$$\frac{\ddot{a}}{a} = \frac{4\pi G}{3}(\rho + 3p)$$

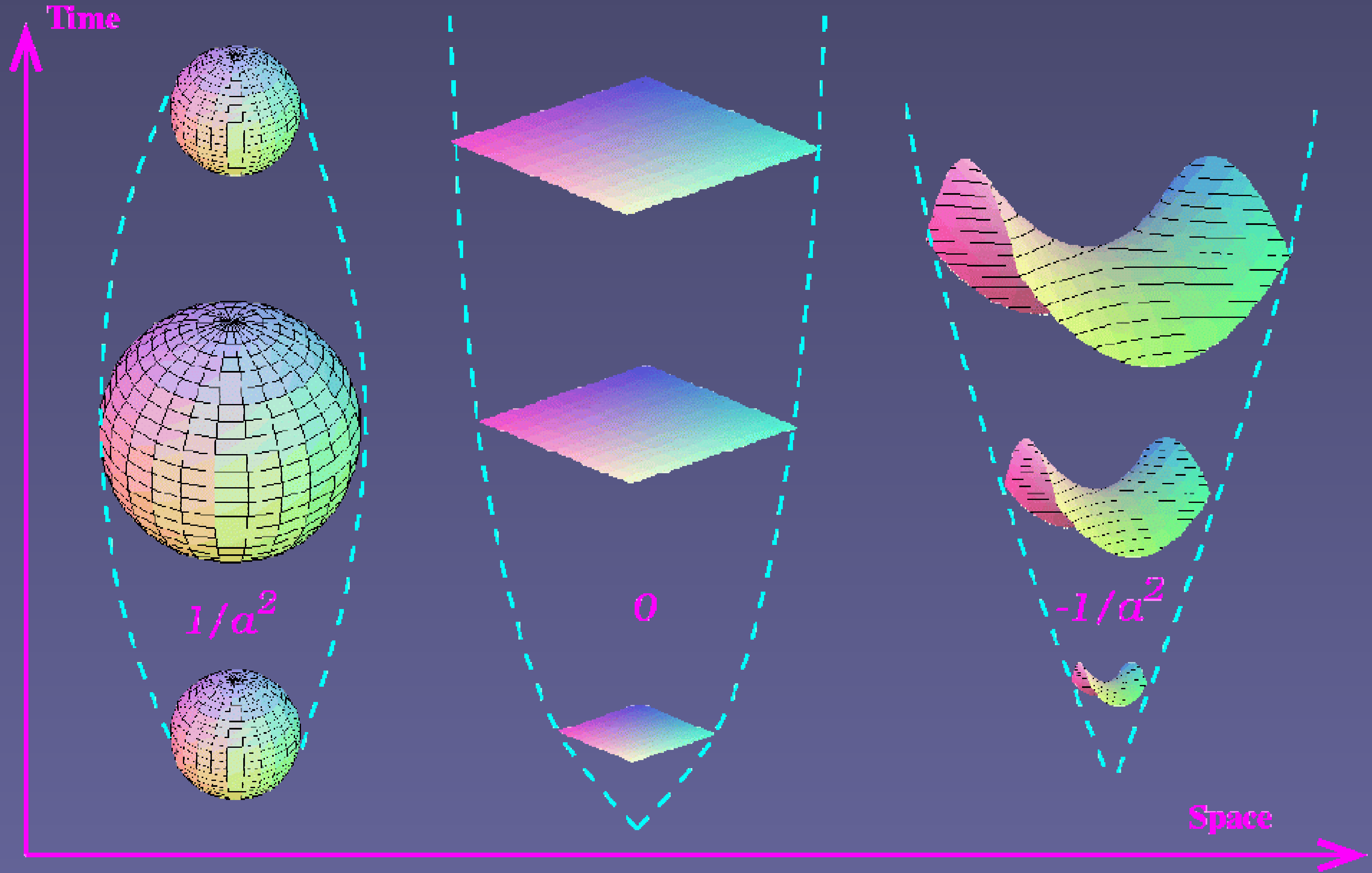
where

$$\dot{\rho} = -3\frac{\dot{a}}{a}(\rho + p) = -3\frac{\dot{a}}{a}\rho(1 + w)$$



$$\ddot{R} = -\frac{GM}{R^2} = -4\pi G\rho R^3 R^2 = -4\pi G\rho R$$

# Homogeneous & Isotropic Universe



# Evolution of Scale Factor

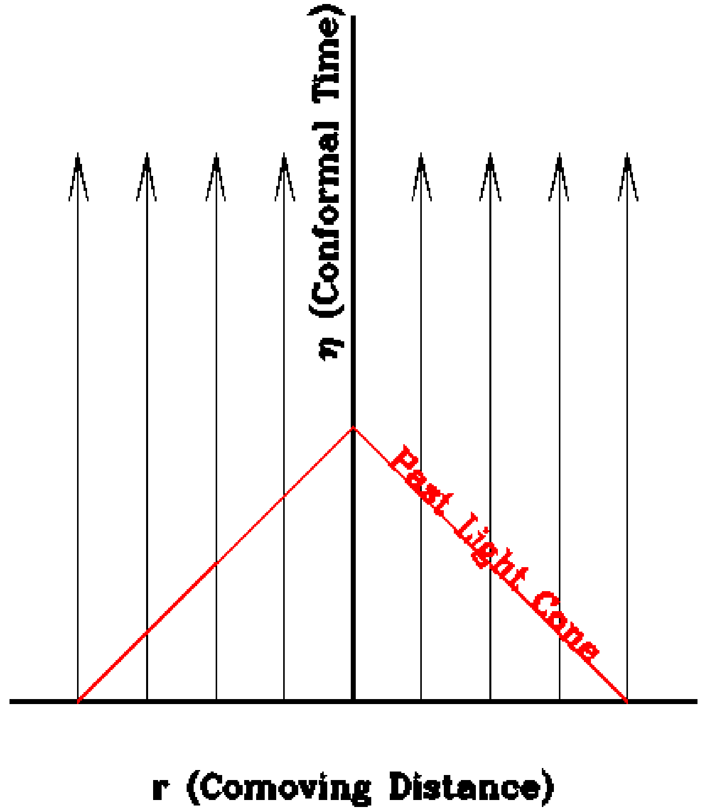
Matter: $w = 0$ $a \propto t^{2/3}$

Radiation $w = 1/3$ $a \propto t^{1/2}$

Vacuum Energy $w = -1$ $a \propto \exp(t)$

$$H^2 = \left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3}\rho$$

# Conformal Structure of Space-time

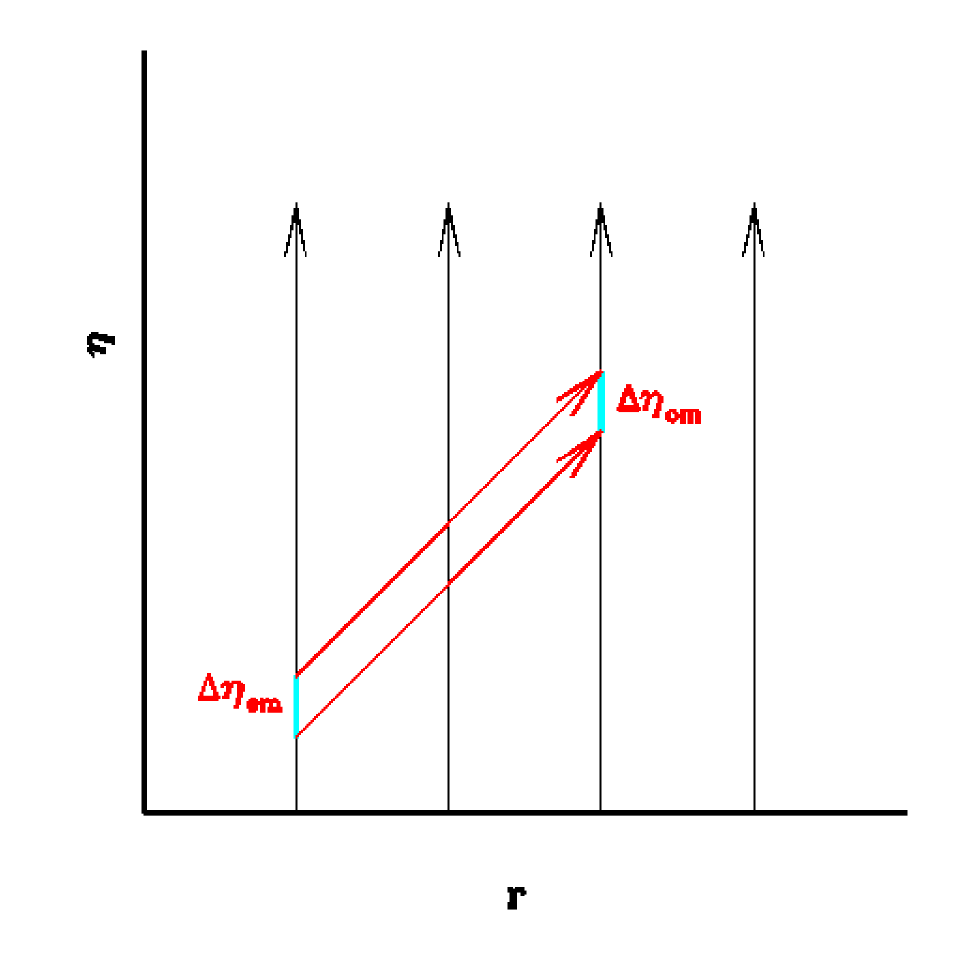



$$\eta = \int \frac{dt}{a(t)}$$

$$(ds)^2 = a(\eta)^2 \left\{ (d\eta)^2 - [(dx)^2 + (dy)^2 + (dz)^2)] \right\}$$

| | | | |
|---|---|---|---|
| matter | $a \propto t^{2/3}$ | $\eta \propto t^{1/3}$ | $a \propto \eta^2$ |
| radiation | $a \propto t^{1/2}$ | $\eta \propto t^{1/2}$ | $a \propto \eta$ |
| VacuumEnergy | $a \propto \exp(t)$ | $\eta \propto (1 - exp(-t))$ | NewCausalStructure |

# Redshift




$$d\eta = \int \frac{dt}{a(t)}$$

$$\Delta\eta_{em} = \frac{\Delta t_{em}}{a(t_{em})}$$

$$\Delta\eta_{ob} = \frac{\Delta t_{ob}}{a(t_{ob})}$$

$$\frac{\Delta t_{ob}}{\Delta t_{emit}} = \frac{a_{ob}}{a_{em}}$$

$$\frac{\lambda_{ob}}{\lambda_{emit}} = \frac{a_{ob}}{a_{em}} = \frac{1+z_{em}}{1+z_{obs}}$$

# Supernova Duration

$$\frac{\Delta t_{ob}}{\Delta t_{emit}} = \frac{a_{ob}}{a_{em}}$$

Goldhaber et al. (2001)
ApJ 558, 359

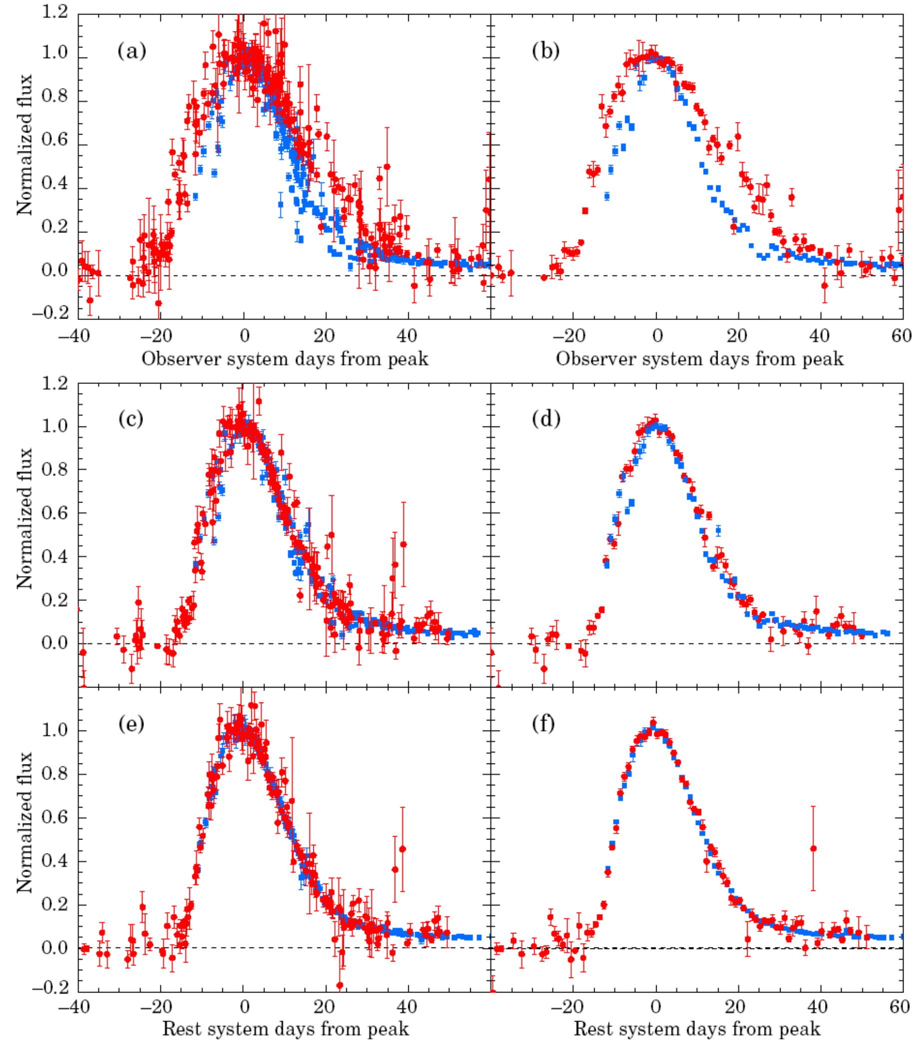


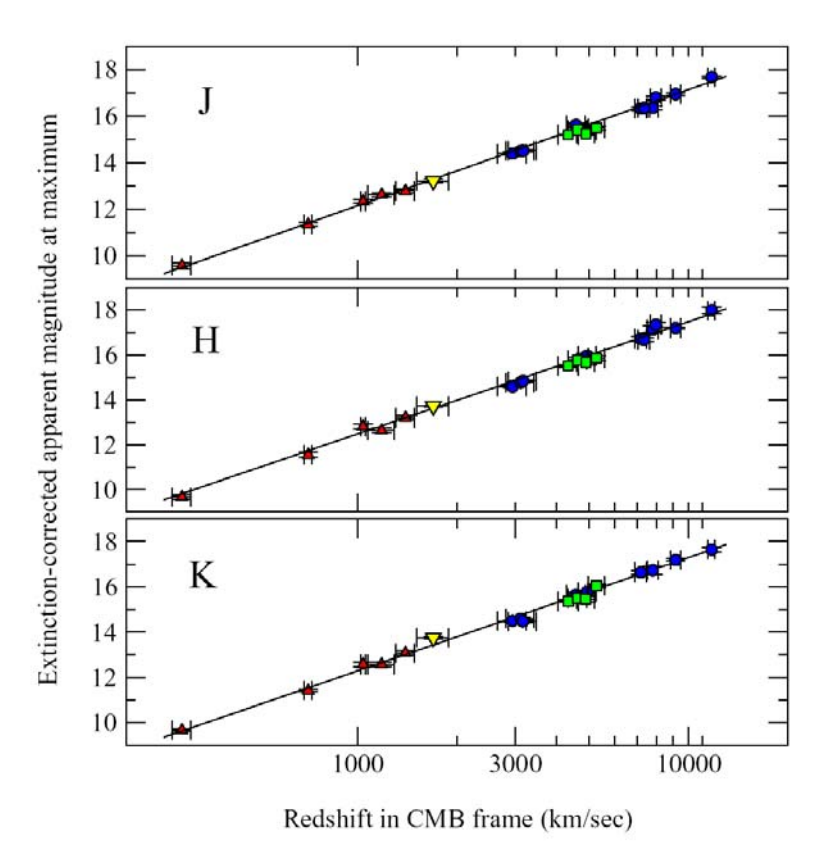

Fig. 1.— (a) The photometry points for the 35 SCP (full red circles) and 18 Calán/Tololo SNe (blue squares), fitted to Parab-18 with the maximum flux normalized to unity and the time of maximum adjusted to zero in the observer system. (b) shows the same data as in (a) averaged over one-day intervals and over each set of SNe. (c) and (d) show the same data, transformed to the rest system. In (e) and (f) the time axis for each photometry point is additionally divided by the corresponding stretch factor $s$.

# Hubble Law

$$\int d\eta = \int \frac{dt}{a(t)} = \int \frac{da}{a\dot{a}} = -\int \frac{dz}{H(z)}$$

$$\Delta\eta = \int_0^z \frac{dz}{H(z)} = \Delta z / H_0$$

$$\Delta z = \frac{v}{c} = H_0 \Delta\eta$$




Krisciunas, et al (2004) AJ 128, 3034

# Rulers and Standard Candles

Luminosity Distance

$$d_L = \Delta\eta(1+z)$$

$$F = \frac{L}{4\pi d_L^2}$$

Angular Diameter Distance

$$d_A = \frac{\Delta\eta}{1+z}$$

$$\Delta\Theta = \frac{x}{d_A}$$

# Flat M.D. Universe

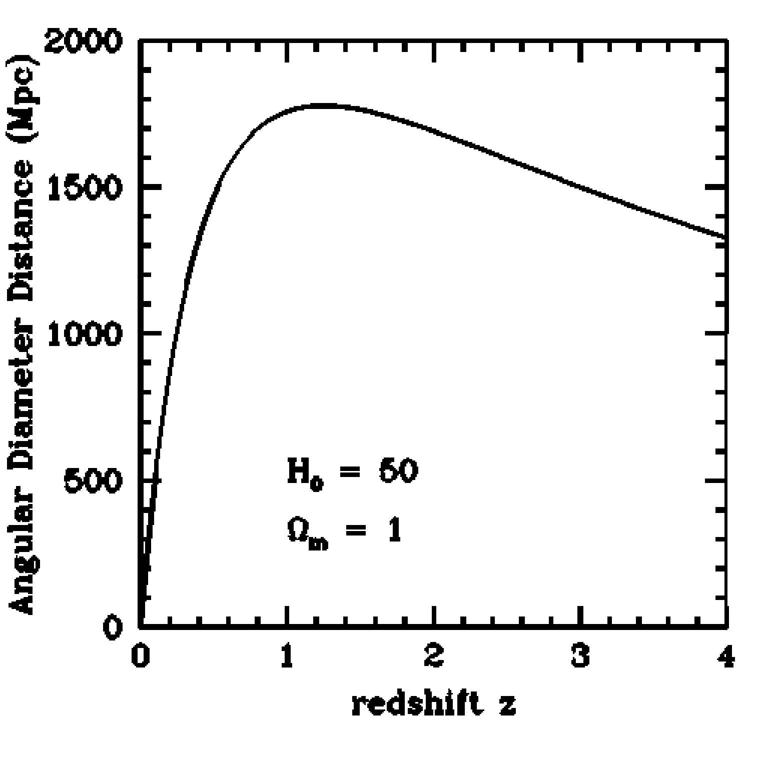
$$a(t) = (t/t_0)^{2/3}$$

$$\frac{c\eta}{2H_0} = a^{1/2} = \frac{1}{\sqrt{1+z}}$$

$$\Delta\eta = \frac{2H_0}{c}\left[1 - \frac{1}{\sqrt{1+z}}\right]$$

$$D_A = \frac{\Delta\eta}{1+z} = \frac{2H_0}{c}\left[\frac{1}{1+z} - \frac{1}{(1+z)^{3/2}}\right]$$

$$D_L = \Delta\eta(1+z) = \frac{2H_0}{c}\left[1+z-\sqrt{1+z}\right]$$




D = 1500 Mpc for z > 0.5

# Measuring Distances

$$\delta\eta = c\int \frac{dz}{H(z)} \simeq \frac{c}{H_0}\left(z - (1+q_0)\frac{z^2}{2} + \dots\right)$$

$$q_0 = \frac{\Omega_m}{2} - \Omega_\Lambda$$

# Closed Universes

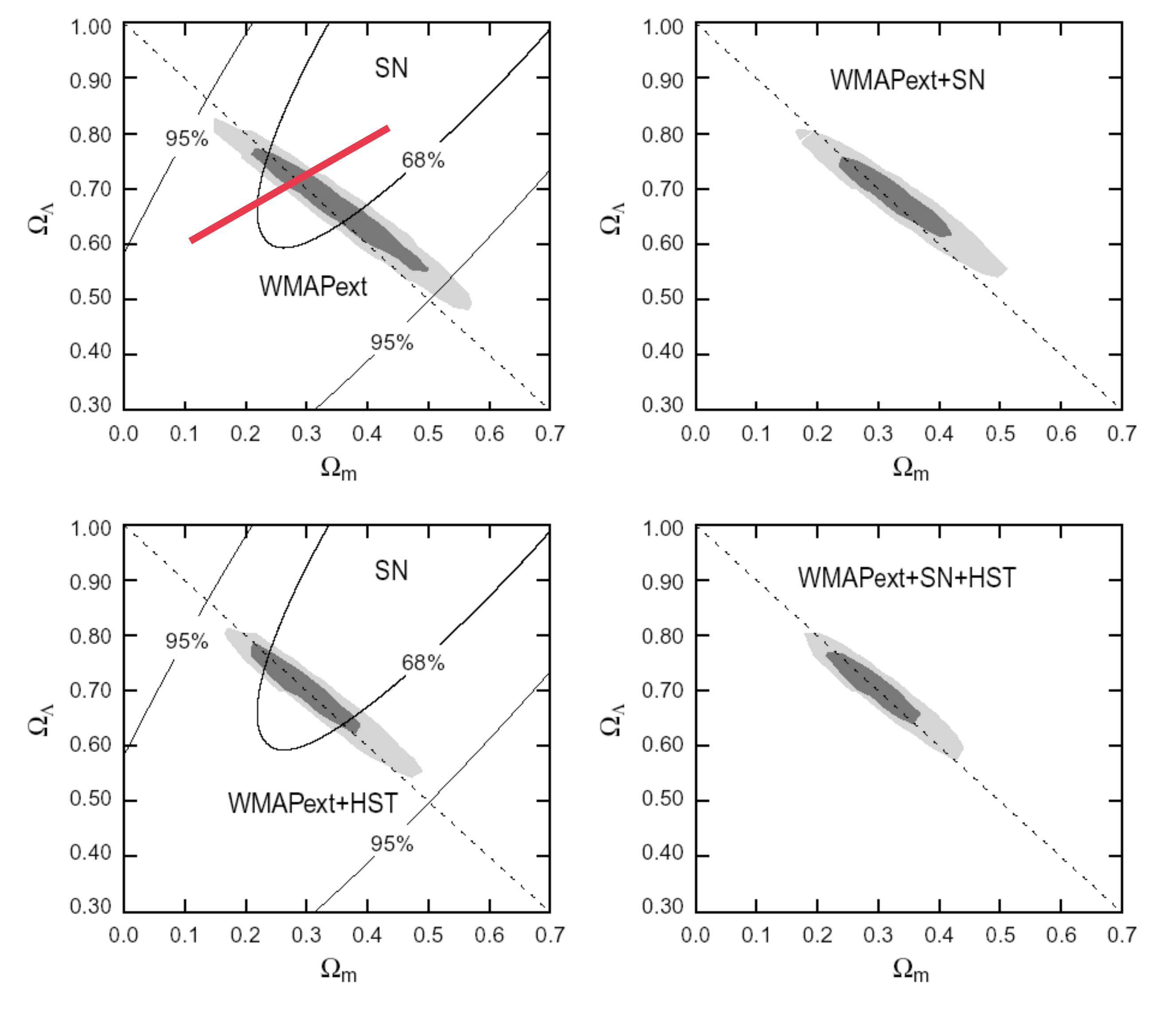
$$(ds)^2 = -(dt)^2 + a^2(t)\left[(dr)^2 + \sin^2(r/R_c)(d\Omega)^2\right]$$

$$H^2 = \left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G\rho}{3c^2} + \frac{c^2}{R_c^2 a^2}$$

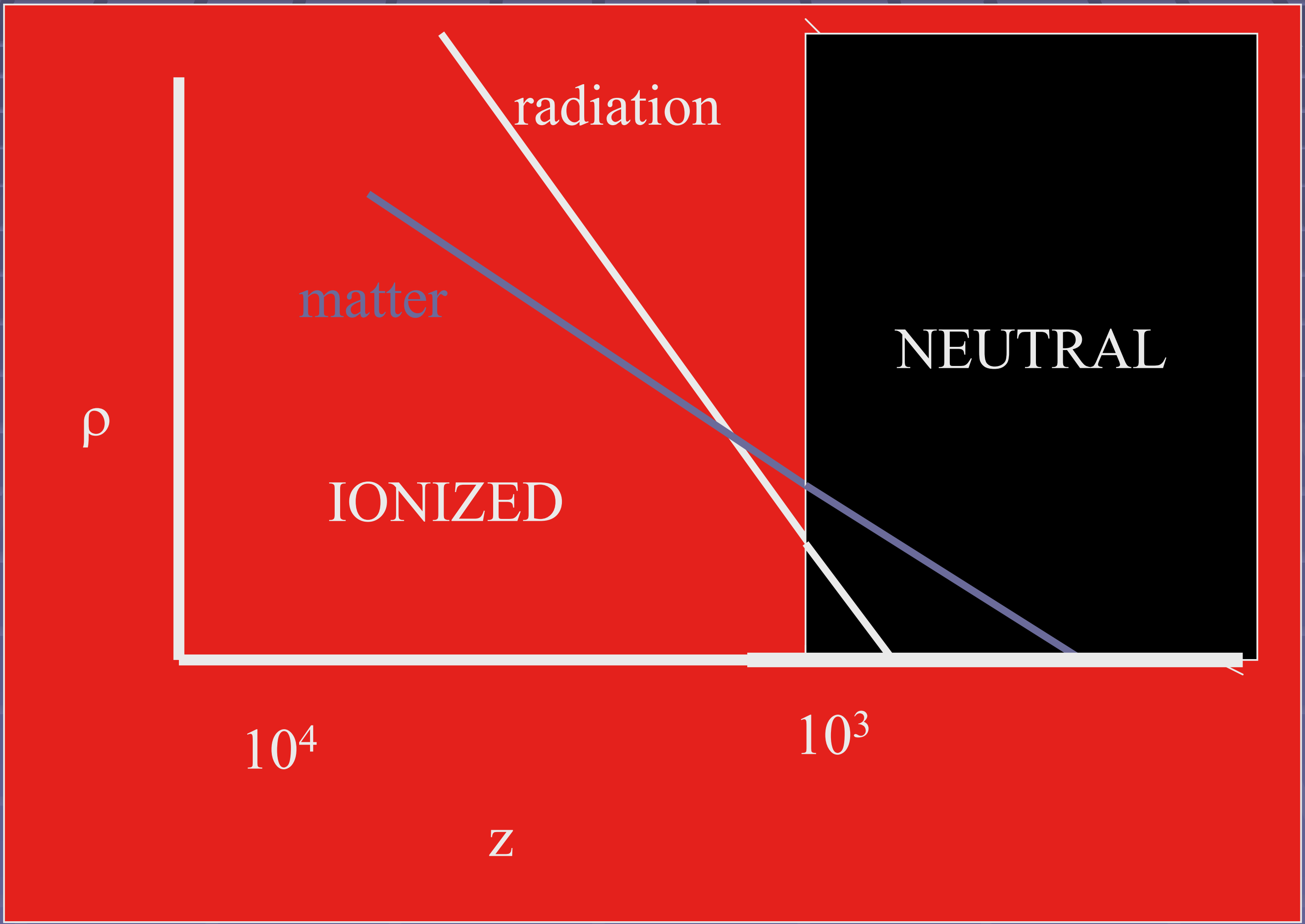
$$d_L = R_c(1+z)\sin\left(\frac{\eta}{R_c}\right)$$

$$d_A = d_L/(1+z)^2$$

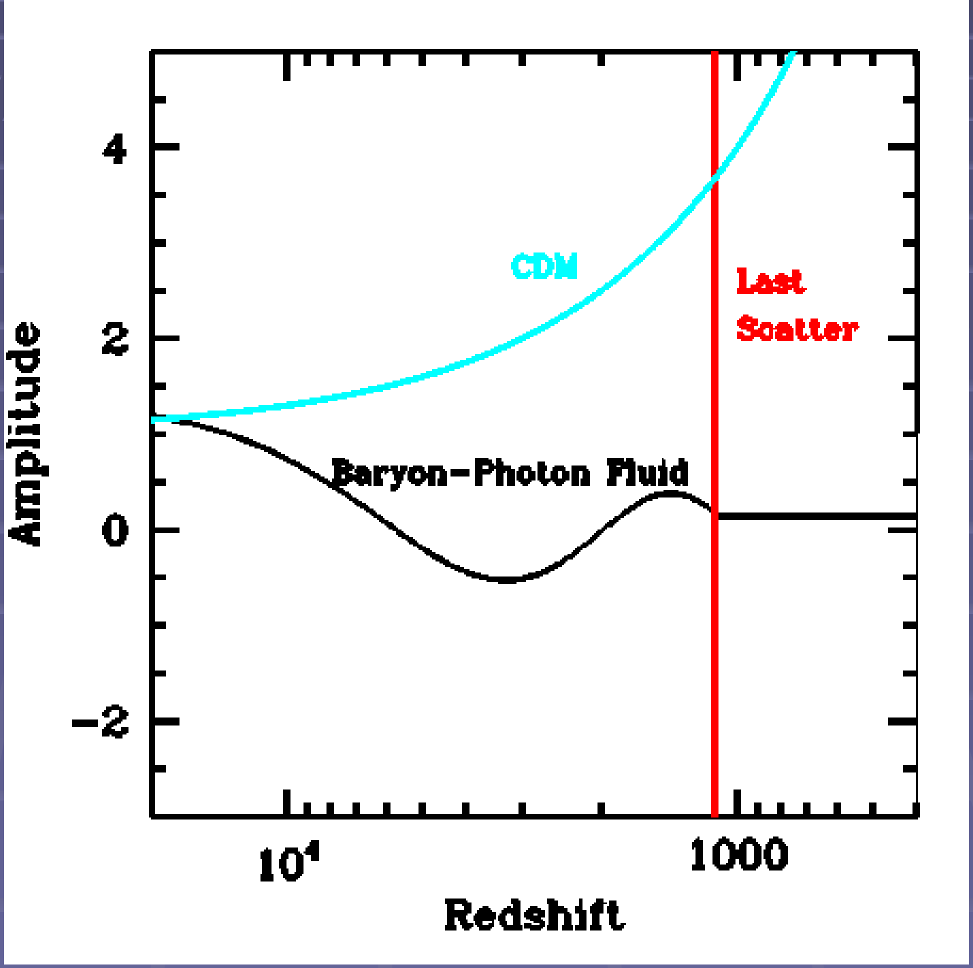
# Distance Measurements



# Thermal History of Universe



# Evolution of Linear Fluctuations



# Structure Formation

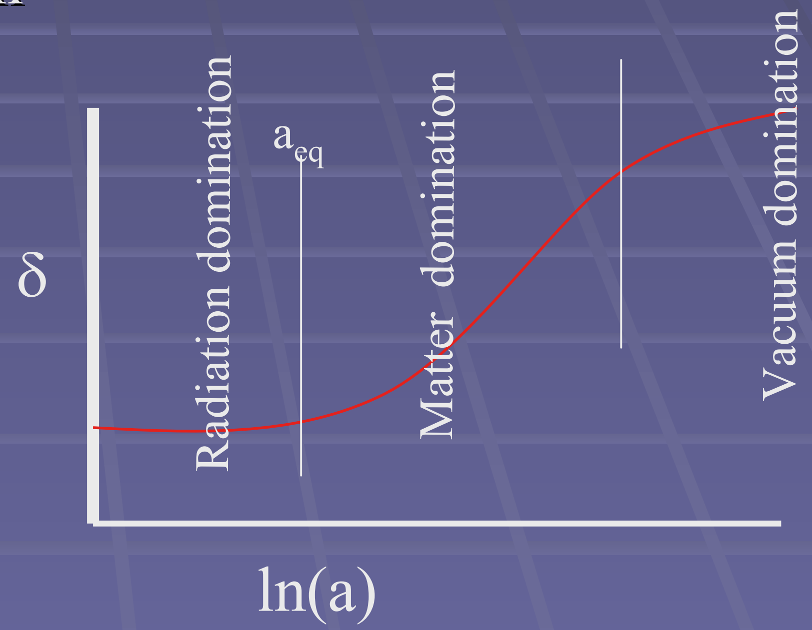
$$\rho(\vec{x},t) = \frac{\rho_0}{a(t)^3}\left[1+\delta(\vec{x},t)\right]$$

$$\delta(\vec{x},t) = \int d^3x \exp(i\vec{k}\cdot\vec{x})\delta(\vec{k})D(k,t)$$

Evolution inside the horizon

$$\ddot{\delta} + \frac{\dot{a}}{a}\delta = 4\pi G\rho\Omega_m\delta$$

$$\delta \propto \left(1+\frac{3}{2}\frac{a}{a_{eq}}\right)$$



# Potential Fluctuations

$$\nabla^2\phi = 4\pi G a^2(\rho - \bar{\rho}) = 4\pi G\frac{\rho_0\Omega_m\delta(\vec{x},t)}{a}$$

$$\phi(\vec{k},t) = 6k^2\eta^2\delta(\vec{k},t) \propto \frac{\delta}{a}$$

$$\int_{k_0}^{2k_0} d^3k|\phi(k.0)|^2 = \int d\ln k k^3|\phi(k,0)|^2 = \text{Constant}$$

$$|\phi(k,0)|^2 = \frac{C}{k^3}$$

$$|\delta(k,\eta)|^2 = 36C\eta^4 k$$

$$P(k,0) \propto k$$

"Gauge Artifact"

# Transfer Function

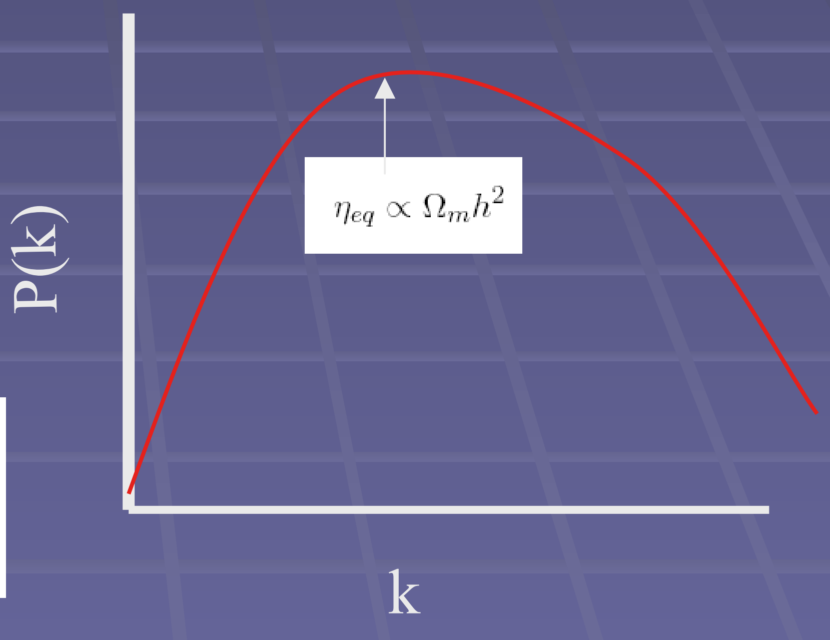
$$\phi(k,t) = \phi(k,0)T(k,t)$$

Matter domination ($a > a_{eq}$) OR $kt << 1$ ⇒ $\Phi$ = Constant

Radiation Domination ($a < a_{eq}$)

$$\Phi(t) = \frac{\delta}{a} \propto \frac{1}{a}$$

$$T(k,t) = \frac{1}{1 + k^2 \eta_{eq}^2}$$



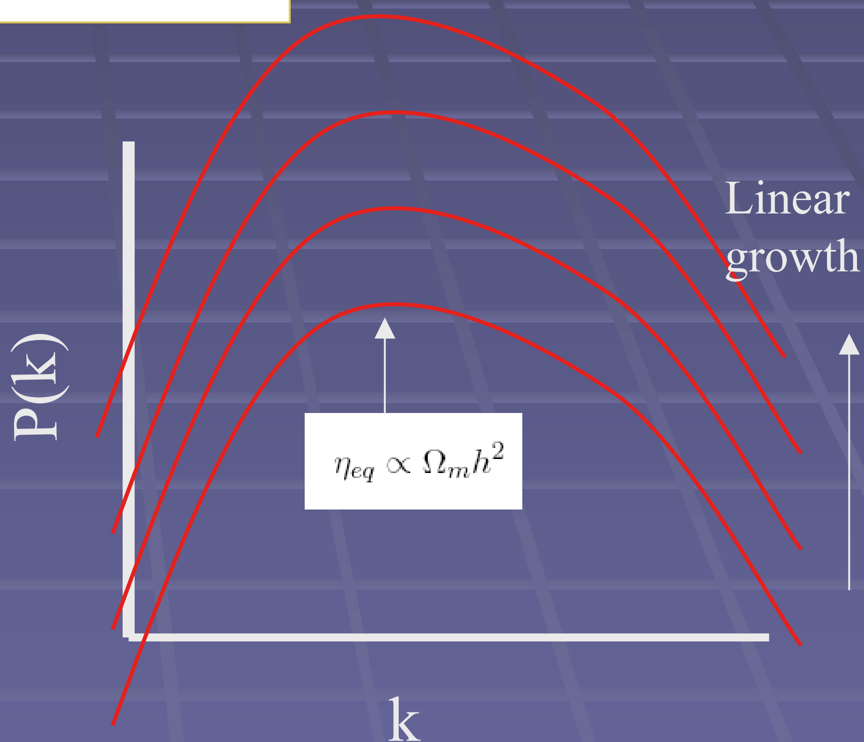
# Evolution of Density Fluctuations

$$P(k,t) = P(k,0)(T(k,t))^2 \propto \frac{k}{(1+k^2\eta_{eq}^2)^2} D^2(a)$$

Linear growth

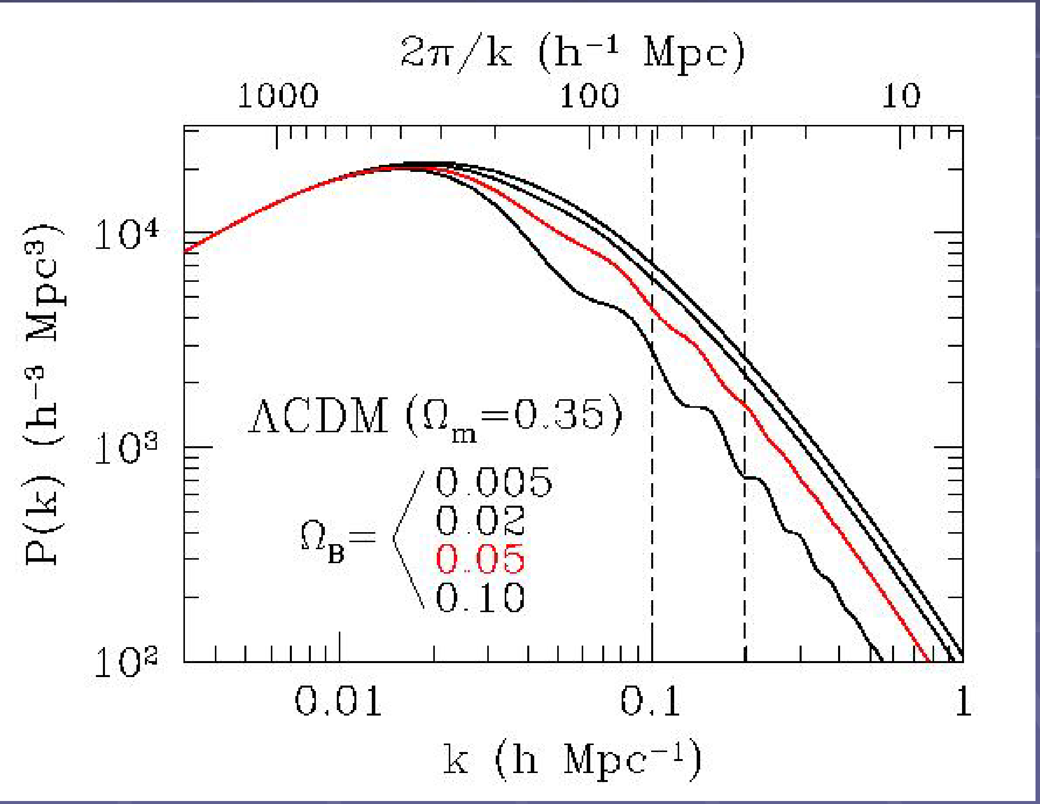
Initial Conditions

Evolution during matter/radiation



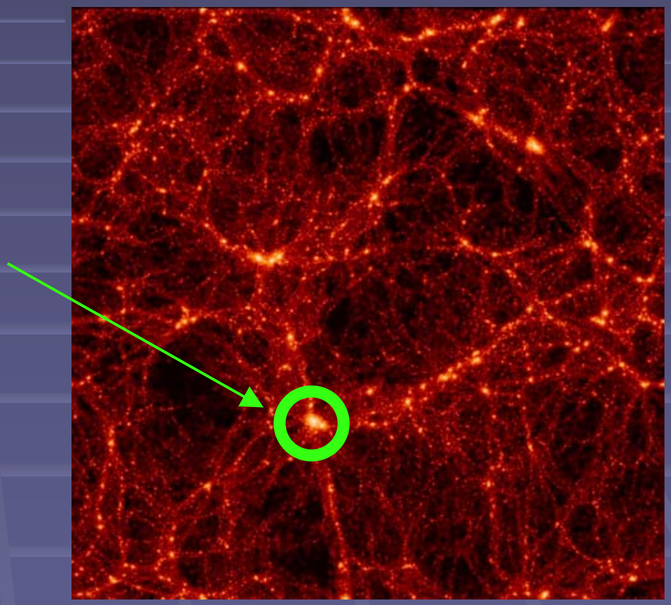
# Baryon Wiggles

- During today's lecture, I have assumed that the radiation was distributied uniformly.
- On Wednesday, I spent the day discussing variation in radiation density (CMB fluctuations)
- If we study the full baryon/photon/dark matter system, the power spectrum is slightly modified

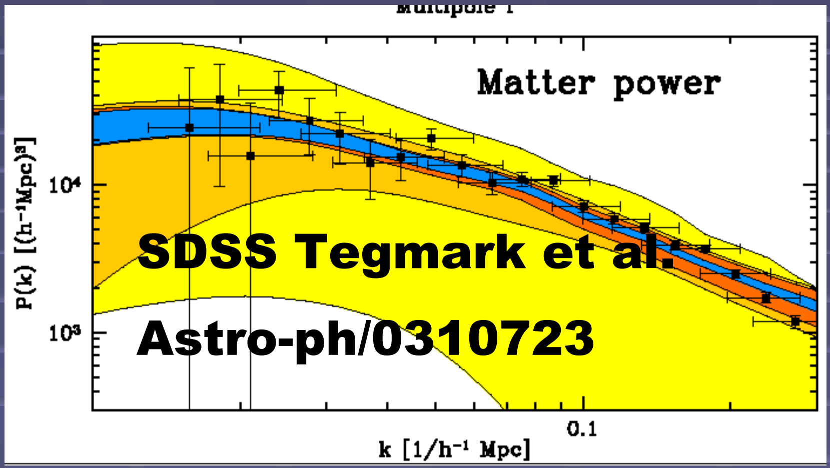


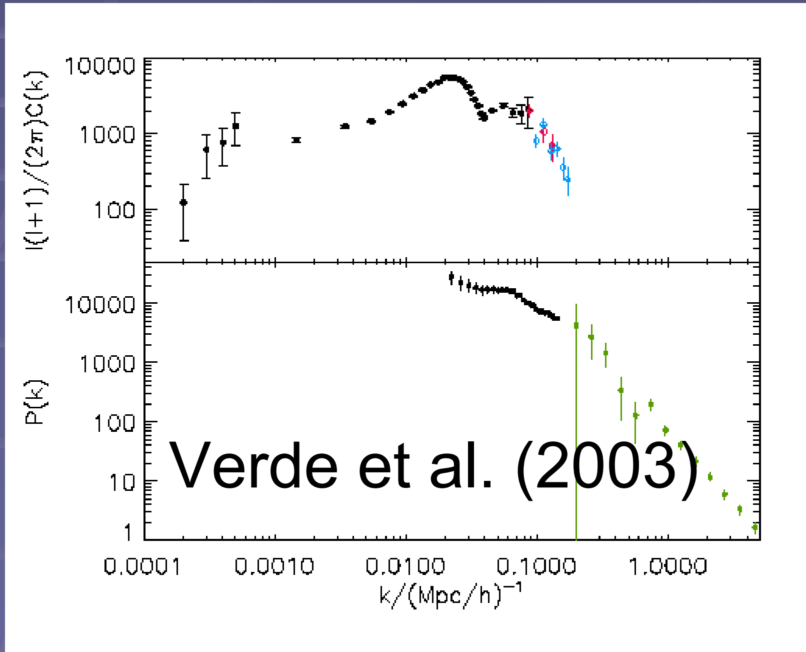
# Numerical Simulations

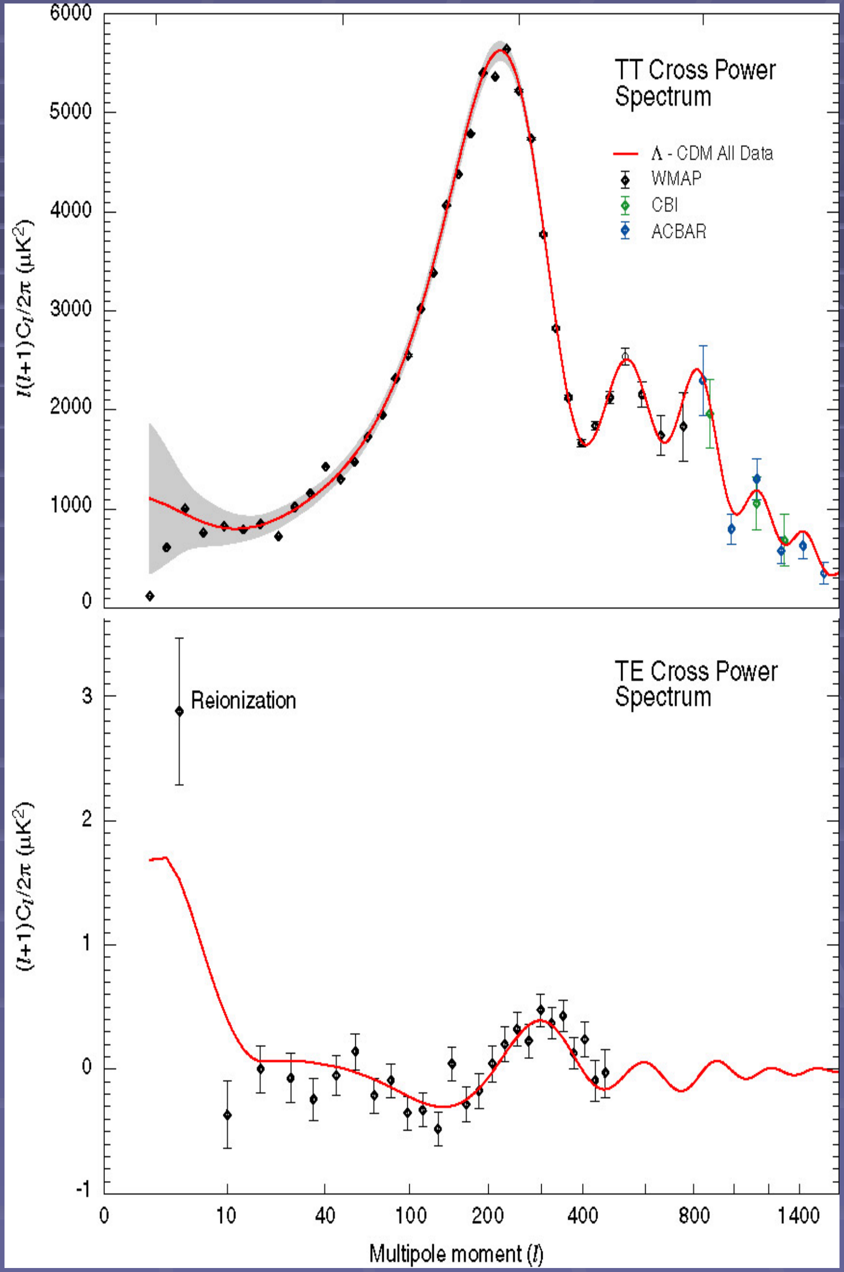
- Start with linear theory
- Gravitational dynamics
- Additional physics
  - Hydrodynamics
  - Cooling and Energy Input
  - Parameterized Star Formation
- Halo Models
  - Identify dark matter halos and study properties



# It Works!


Matter power
P(k) [(h⁻¹Mpc)³]
k [1/h⁻¹ Mpc]
SDSS Tegmark et al.
Astro-ph/0310723


l(l+1)/(2π)C(k)
P(k)
k/(Mpc/h)⁻¹
Verde et al. (2003)


TT Cross Power Spectrum
Λ - CDM All Data
WMAP
CBI
ACBAR
TE Cross Power Spectrum
Reionization
Multipole moment (l)

# Consistent Parameters

| | WMAP+CBI+ ACBAR | All CMB(Bond) | CMB+ 2dFGRS | CMB+SDSS (Tegmark) |
|---|---|---|---|---|
| $\Omega_b h^2$ | .023 $\pm$ .001 | .0230 $\pm$ .0011 | .023 $\pm$ .001 | .0232 $\pm$ .0010 |
| $\Omega_x h^2$ | .117 $\pm$ .011 | .117 $\pm$ .010 | .121 $\pm$ .009 | .122 $\pm$ .009 |
| h | .73 $\pm$ .05 | .72 $\pm$ .05 | .73 $\pm$ .03 | .70 $\pm$ .03 |
| $n_s$ | .97 $\pm$ .03 | .967 $\pm$ .029 | .97 $\pm$ .03 | .977 $\pm$ .03 |
| $\sigma_8$ | .83 $\pm$ .08 | .85 $\pm$ .06 | .84 $\pm$ .06 | .92 $\pm$ .08 |

# Is There More Physics?

- More complicated initial conditions
  - P(k) could be more complicated
  - Isocurvature fluctuations of various kinds (cosmic strings, dark matter/photon ratio fluctuations)
  - Non-Gaussianity
  - Finite Universe (or Infinite Open Universe)
- Richer physics for constituents of universe
  - Neutrino mass
  - Dark matter interaction
  - Dark energy evolution
  - Deviations from general relativity

# Open Questions

- What is the dark matter?
  - Experimental searches for supersymmetric dark matter; axions
  - LHC and supersymmetry
- What are the initial conditions?
  - CMB, Large-scale structure measurements, etc.
- What is the dark energy?  Is it new physics?