REDSHIFT

Always true : $\lambda_{obs} = \lambda_{rest}(1+z)$ or $z = \dfrac{\lambda_{obs} - \lambda_{rest}}{\lambda_{rest}}$

this is equivalent to $z = \dfrac{\frac{c}{\nu_{obs}} - \frac{c}{\nu_{rest}}}{\frac{c}{\nu_{rest}}} = \dfrac{\nu_{rest}}{\nu_{obs}} - 1 = \dfrac{\nu_{rest} - \nu_{obs}}{\nu_{obs}}$

DON'T USE :

Some radioastronomers (at low z) $z_{radio} = \dfrac{\nu_{rest} - \nu_{obs}}{\nu_{rest}} \neq z_{opt}$

— x —

VELOCITY

low velocity (redshift) approximation $1+z = 1 + \frac{v}{c}$ DOPPLER!

Special Relativity (approximation) $1+z = \left(\dfrac{1+v/c}{1-v/c}\right)^{1/2}$ DOPPLER!

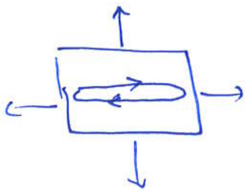
General Relativity $1+z = \dfrac{R(t_{obs})}{R(t_{emit})}$ SPACE EXPANSION

where R is the scale factor. R(t) depends on Cosmolog

$H = \dfrac{\dot{R}}{R}$ and $H_0 = \dfrac{\dot{R}(t_{now})}{R(t_{now})}$

— x —

Expanding box :



1ⓐ for a particle : momentum $\propto \frac{1}{R}$ where R is scale factor (size of box) →

equivalent in an expanding universe.

if $v \ll c$ mass = constant . So eqn ① ⇒ $v \propto \frac{1}{R}$

or $v = v_0(1+z)$

$E \propto v^2$ ⇒ energy = energy now × $(1+z)^2$

1ⓑ for a relativistic particles $(v \sim c)$ or ~~light~~ photon $(v=c)$

$E = pc$. So eqn ① ⇒ energy = energy now × $(1+z$

for light energy $\propto$ freq so freq $\propto (1+z)$ → OVER

2(a) collection of particles: (in an expanding box or universe) = gas

particles collide. In between collisions they lose energy

$E = KT$

i.e. gas temperature $T \propto \frac{1}{R^2}$ or $T = T_0 (1+z)^2$

2(b) collection of photons (photon gas) in an expanding box

if in thermal equilibrium → thermal or black body radiation.

Temp of thermal radiation is the measure of the average energy of ph

Since photon energy $\propto \frac{1}{R}$

$T = T_0 (1+z)$

$\cos(A+B) = \cos A \cos B - \sin A \sin B$

$$V_{RF} = A(t) \cos(\omega_0 t + \phi(t))$$

$$V_{LO} = A_{LO} \cos(\omega_{LO} t)$$

$$V_{out} = V_{RF} * V_{LO} = \frac{A(t) A_{LO}}{2} \Big\{ \cos\phi \left(\cos(\omega_{LO}+\omega_0)t + \cos(\omega_{LO}-\omega_0)t \right)$$
$$- \sin\phi \left(\sin(\omega_{LO}+\omega_0)t + \sin(\omega_{LO}-\omega_0)t \right)$$

$$= \frac{A(t) A_{LO}}{2} \Big\{ \cos((\omega_{LO}+\omega_0)t + \phi(t)) + \cos((\omega_{LO}-\omega_0)t + \phi(t))$$

i.e. 2 new freq: LO+RF and LO−RF

upper sideband ← LO+RF; lower sideband ← LO−RF

∴ use high pass or low pass filter to select one sideband.

note that LO can be below or above the RF

For a given LO, energy at LO ± IF is converted to the same IF frequency. PROBLEM!

$\boxed{\text{DSB or SSB mixers.}}$

e.g. RF = 1 GHz = 1000 MHz IF = 100 MHz

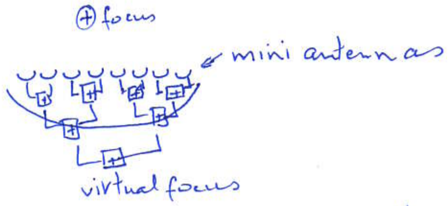
LO = 900 MHz

Then RF at 1 GHz and noise/signal at 800 MHz will both be converted to the same IF

Can prefilter to seperate the two SBs and can then get a SSB receiver.

additional notes to Uson lectures

(1) Interferometry



V = ~~to~~ voltage of signal

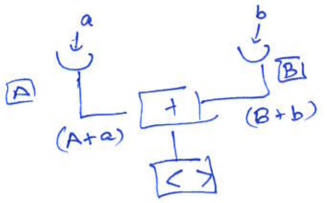
$V \propto E \propto \sqrt{I}$ ← Intensity

voltage (V); elec field (E)

i.e. $V_1 V_2 \propto$ Intensity I_ν units Watt m^{-2} Hz^{-2} $ster^{-2}$

BASIC

(2) Adding Interferometer Signal a, b Noise A, b ← NAIC lectures



before detector $(A+a)+(B+b)$

After detector $[(A+a)+(B+b)]^2$

Time average of uncoorr quantities $\to 0$

$<>^2 = A^2 + B^2 + a^2 + b^2 + 2a.b.$

($2a.b$) ← this what we want

BASIC

(3) Mult Interferometer $(A+a)(B+b) = A.B + A.b + a.B + a.b$

$< > = a.b!$

∴ ie Multiplying + Averaging = CORRELATION

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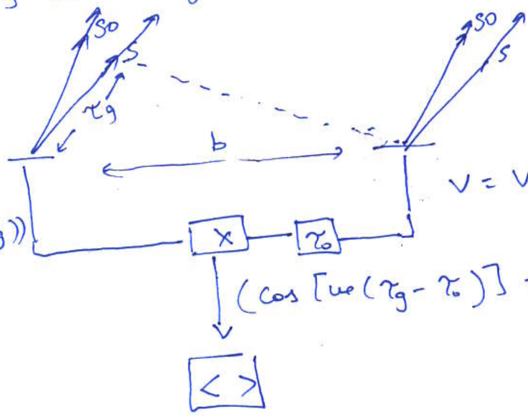
(4) "Real" Stationary RF Interferometer with inserted time delay.

S_0 = reference dir

S = general dir

$\tau_g = b.s/c$

$\tau_0 = b.s_0/c$



$V = V_2 \cos[\omega(t-\tau_0)]$

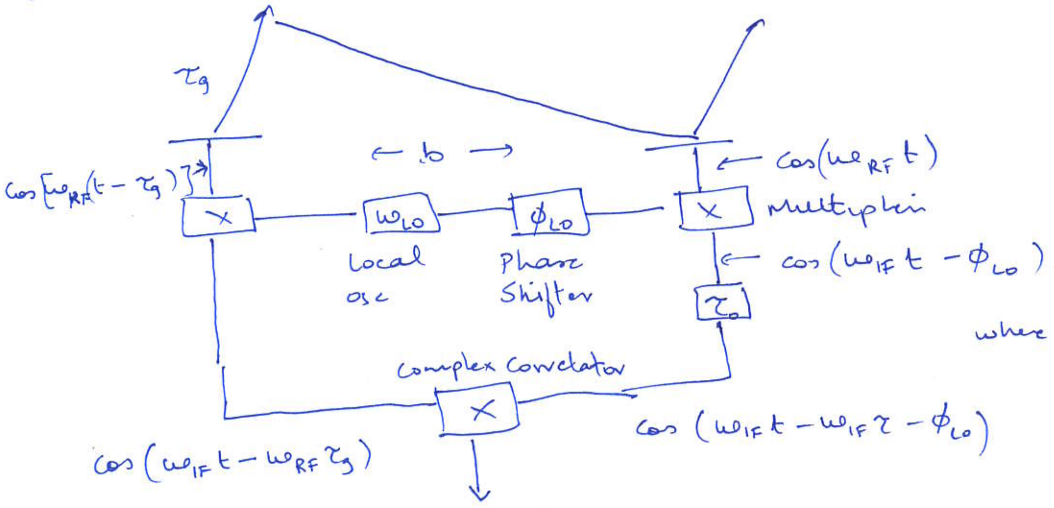
$V = V_1 \cos(\omega(t-\tau_g))$

$(\cos[\omega(\tau_g-\tau_0)] + \cos[2\omega t - \omega(\tau_g-\tau_0)])/2$

$\frac{V_1V_2}{2}\cos(\omega(\tau_g-\tau_0)] = \frac{V_1V_2}{2}\cos[2\pi\nu b.(s-s_0)/c]$

Downconversion

→ we have to | because ~~can~~ not many components work at many RF's



where $\omega_{RF} = \omega_{LO} + \omega_{IF}$)

$$V = e^{-i(\omega_{RF}\tau_g - \omega_{IF}\tau_0 - \phi_{LO})}$$

This will be identical to previous RF interferometer if the phase in the exponential is equal to $\omega_{RF}(\tau_g - \tau_0)$

This can be done by adjusting LO phase : $\phi_{LO} = \omega_{LO}\tau_0$

Duh! why was this neccesary? ∵ τ_0 added in IF portion of signal path.

coherence : stability & predictability of the phase

spatial & temporal coherence

Spatial coherence : Young's double slit → change d.

If the pattern dies out after the 1st fringe, the temporal coherence $\sim 1\lambda$

i.e. for laser light : interference bands are very wide

for incoherent radiation : if intensity patterns could be detected within its very short temporal correlation length, then the fringe pattern would extend to infinity. Over many events, averaging flattens out the extended pattern

$$\psi_A(x,t) = A\, e^{i\,(k(\omega)t - \omega t - \delta_A(t))}$$

$$\psi_B(x,t) = B\, e^{+i\,(k(\omega)x - \omega t - \delta_B(t))}$$

$\delta(t) = \delta_A(t) - \delta_B(t)$ if this is ~~small~~ during a sampling time, then coherent. If $\sim 2\pi$ during sampling time, then incoherent.

— x —

Intensity of a black-body radiator is given by the Planck formula

$$B(\nu) = \frac{2h\nu^3}{c^2}\frac{1}{e^{h\nu/kT} - 1}$$ for ~~low T~~ low ν or high T,

Raleigh Jean's law : $h\nu \ll kT \Rightarrow e^{h\nu/kT} \sim 1 - \frac{h\nu}{kT}$

$$\Rightarrow B(\nu) = \frac{2kT\nu^2}{c^2}$$

extended black body with temp distr $T(\theta,\phi)$ is $S = \frac{2k\nu^2}{c^2}\int T(\phi,\theta)\, d\Omega$

even if not black body, use brightness temp : $T_b(\theta,\phi) = \frac{c^2}{2k\nu^2} B(\theta,\phi)$ where $B(\theta,\phi)$ is the surface brtness of the extended source. If black body : $T_b = T$, else is the T a black body would need to have the same flux at the same freq.